# E ANCIENT CUBIT

AND

# WEIGHTS AND MEASURES

SIR CHARLES WARREN

PALESTINE EXPLORATION FUND

# THE ANCIENT CUBIT

AND

# *OUR WEIGHTS AND MEASURES*

BY

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'A Budget of Paradoxes.' De Morgan.

'Greek Geometry.' Allman.

'The Pyramid of Gizeh.' Petrie.

'A Short History of Mathematics.' Ball. 1893.

Josephus, Plutarch, Herodotus, Polybius, and Strabo.

'The Great Pyramid.' Taylor. 1859.

Whitaker's Almanack. 1900.

Transactions of the Royal Historical Society. 1900, 1901. The Tribal Hidage; Peter's Pence.

'Dictionary of the Bible.' Smith. Articles: Metals; Money; Weights and Measures.

'Dictionary of the Bible.' Hastings. Articles: Babylonia; Money; Weights and Measures.

'Dictionary of Greek and Roman Antiquities.' Smith. Articles: Mensura; Nummus; Pondera; Stater, etc.

'Ancient Weights and Measures.' Hussey.

'Jewish Coinage.' Madden. 1864.

'Handbook to the Bible.' Conder. 1880.

'Our Inheritance in the Great Pyramid.' Smyth. 1890.

'Proceedings of the Society of Biblical Archæology.' 1891 to 1902. The Rhind and Ebers Papyri; Egyptian Weights and Measures, etc.

'Egypt and Syria.' Dawson. 1892.

'A History of Egypt.' Budge. 1902.

'The Dawn of Civilization.' Maspero. 1897.
'Records of the Past.' Rawlinson. 1873.
'Nineveh and its Remains.' Layard. 1849.
'Origin of Currency and Weight Standards.' Ridgeway, 1892.
'The Evolution of Modern Money.' Carlile. 1901.
'A Bushel of Corn.' A. Stephen Wilson. 1883.
'Ancient Coins, etc.' Arbuthnot. 1754.
'Connexion of Coins.' Roman; Saxon; English. Clark. 1767.
'Universal Cambist.' Kelly. Vols. i. and ii. 1821.
'Coins of the Realm.' Liverpool. 1805.
'Foreign Measures.' Carrington. 1864.
'Measures, Weights, and Money.' Pasley. 1834.
'Tables of Weights and Measures.' Buchanan. 1829.
'The Roman Foot and Denarius.' Greaves. 1647.
'Handbook to Roman Coins.' Madden. 1861.
'Encyclopædia Britannica.' Weights and Measures.
'Penny Cyclopædia.' Weights and Measures.
Palestine Exploration Fund Quarterly Statement. 1879: Measures, by Beswick. 1897 to 1902: Weights and Measures, by Watson, Conder, and Warren.
'Bimetallisme chez les Hébreux.' De Salignac Fénélon.

# PREFACE

In putting this volume before the public, I beg most cordially to acknowledge my obligations to many friends for the valuable assistance they have given me. More particularly my thanks are due to Colonel C. R. Conder, LL.D., R.E., for examining and criticising the portions with which he is most familiar—viz., those dealing with weights and measures of the East. To Lieutenant-Colonel Elliot, R.E., and Captain F. Warren, R.A., I am much indebted for the trouble they have taken in assisting me to render many of the intricate arithmetical calculations clearer than they would otherwise have been, and for having looked through most of the proofs; and to the Rev. W. J. Foxell, M.A., my thanks are due for looking through the proofs from a more general point of view. From Colonel Bolland, R.E., Colonel Watson, R.E., the Rev. J. E. Hanauer, Dr. T. Chaplin, and Mr. H. J. Chaney I have received valuable information regarding the weight of the grains of barley and wheat in ancient and modern times.

CHARLES WARREN.

Ramsgate,
*December*, 1902.

# ERRATA.

Page 9, line 15, *for* "17" *read* "16·5".

Page 9, line 24, *for* "$857\frac{3}{6}$" *read* "$857\frac{3}{8}$".

Page 10, line 10, *for* "$1\frac{1}{4}$" *read* "$\frac{1}{6}$".

Page 15, last line but one, *for* "$\left\{{}^{3}\left(\frac{4}{3}\right)^{2}\right\}$" *read* "$\left\{3\left(\frac{4}{3}\right)^{2}\right\}^{2}$".

Page 16, line 9, *for* "$\frac{16}{9}$" *read* "$\left(\frac{4}{3}\right)^{2}$" or "$\frac{16}{9}$".

Page 20, line 6, *after* "$\pi=\frac{3r+1}{r}$" *add* "as a foundation."

Page 31, line 22, *for* "$\frac{1}{7\times 8\times 9\times 11}$" *read* "$\frac{1}{7\times 8\times 97}$".

Page 34, line 18, *for* "the root" *read* "the cube root."

Page 35, line 6, *for* "17,500" *read* "17·5".

Page 36, line 8, *for* "$\left(\frac{53}{36}\right)$" *read* "$\left(\frac{53}{36}\right)^{3}$".

Page 38, line 1, *for* "Table (I.)" *read* "Table (Iᴀ.)".

Page 40, line 7, *for* "$\frac{1}{10}(17{,}500)^{\frac{5}{2}}$" and "$\frac{1}{10}(17{,}500)^{\frac{1}{6}}$" *read* "$\left(\frac{17{,}500}{(10)^{3}}\right)^{\frac{2}{5}}$" and "$\left(\frac{17{,}500}{(10)^{3}}\right)^{\frac{1}{6}}$".

Page 40, line 9, *for* "1,749·34" *read* "17,493·4".

Page 40, line 12, *omit* "$\frac{53}{36}$".

Page 40, line 14, *for* "$10)^2$" *read* "$(10)^2$".

Page 48, line 18, *for* "$(c \times \frac{1}{2} c)$" *read* "$(c + \frac{1}{2} c)$".

Page 51, line 23, *for* "400" *read* "420".

Page 53, line 12, *for* "dimension" *read* "dimensions".

Page 55, line 3, *for* "$41\frac{2}{3}$" *read* "$20\frac{11}{18}$".

Page 56, line 1, *for* "36,010" *read* "36·01".

Page 58, line 18. *for* "coin" *read* "weight."

Page 58, line 31, *for* "coins" *read* "weights."

Page 65, line 20, *for* "one-tenth of the gallon" *read* "one-tenth the weight of a gallon of water".

Page 75, line 4, *for* "5,891" *read* "5,844".

Page 76, line 11, *for* "square palm" *read* "palm."

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# INTRODUCTION

In these pages I have endeavoured to indicate the lines on which the early races of Babylonia worked out the original system of weights and measures in use among mankind—a system on which all other systems in use in the world until the end of the eighteenth century are founded.

From the early civilized times of Babylonia and Egypt until after the Middle Ages in Europe there was little progress in arithmetic, notation, and computation until the Arabic numerals were introduced in the fifteenth century; but even up to the sixteenth century (outside Italy) Greek and Roman numerals were principally used, and merchants continued to keep their accounts in Roman numerals until about the year 1550 throughout Europe.

No real progress, however, was made in the system of arithmetical calculations until the beginning of the seventeenth century, when Napier introduced or invented logarithms, and Briggs improved upon his system and also introduced the decimal notation for fractions. Until that time all persons, except an intelligent few, who worked out a system of notation for themselves, had to adhere to the very cumbersome notation of the ancients.

By the eighteenth century mathematicians had assimilated the knowledge from the Arabic and Greek writers, printing had rendered mathematical works accessible to the many, and Newton and Leibnitz had advanced the

power of computation in a remarkable degree. In 1788 Thomas Williams published a work in London *for fixing a universal standard for weights and measures,* and in 1793 Lagrange, as President of the Commission for Reforming Weights and Measures in Paris, proposed the decimal subdivisions, which led to the introduction of the present metrical system, now in use in Europe, and which is the first absolute departure from ancient tradition in such matters.

It is to be noted that until the improvements were made in arithmetical notation between the fifteenth and eighteenth centuries the introduction of the metrical system would have been useless, and even scarcely practicable, and even now there is a decided tendency among mankind to adhere to the old traditional weights and measures as most acceptable for use in everyday life, though for calculations in science, in engineering, in commerce, etc., no doubt the metrical system is greatly to be preferred as more simple and time-saving.

In considering the subject of weights and measures in Europe and the East I have been greatly struck with the tenacity with which mankind has adhered to its weights and measures, handed down through long series of years, showing that there must be something about them which appeals to the human mind and understanding as convenient for ordinary business. Looking at the matter from this point of view, I commenced in 1898 to study the subject from the standpoint of an intelligent ancient groping after light, and I attempted to start away from the beginning of things with no stock in hand except my ten fingers and an *abacus* or *swan-pan,* without knowledge of arithmetic, with no system of notation, and without logarithms —in fact, with nothing except the light of nature.

It was impossible really to divest myself of all I had learnt. In my efforts I soon found that I could only move forward from the beginning on certain lines, and gradually

I found a system presenting itself which I have no doubt was that evolved by the ancients, as I was aided by the ancient works, of which extracts are extant, and by our existing vestiges of the ancient system, preserved so wonderfully in Great Britain.

In my investigations nothing has struck me so much as the remarkable tenacity with which a mixed race like that in Great Britain and Ireland has adhered to ancient weights and measures which have become vitiated or had disappeared from the rest of Europe by the end of the eighteenth century.

In 1899 (July and October) and 1900 (April), I published in the *Palestine Exploration Fund Quarterly* the results of my investigations, showing that the British weights and measures are all drawn from one source—the double cubit cubed of Babylonia—and that the most accurate records of the original measures are to be found in the Great Pyramid of Gizeh, though close approximations can be made from the Egyptian cubit, from the architectural remains all over the world, from extant weights and measures, and coins, although the latter are not of great value as weights unless they have been struck for the double purpose of weight and money.

Finding that during the last three years I have added considerably to my information on the subject, I have now revised the work, and am publishing it in the following pages in the hope that I may be able to demonstrate how certain it is that all weights and measures up to the end of the eighteenth century are derived from one source, and how remarkably these weights and measures have been preserved in our British Empire.

I therefore now introduce the subject by showing briefly how the connection between the weights and measures of Great Britain, Babylon and Egypt came about.

The ancients originally made their measurements in palms and in cubits of 6 palms, but in process of time they were

forced by the *science of numbers* to use a cubit of 7 palms for building purposes, in order to avoid unmanageable fractions, and to adhere to whole numbers as nearly as was practicable; for in those days they had but the very embryo of a decimal notation, and could not express fractions as decimals.

During their investigations (whilst attempting to square the circle) they hit upon the number of 44 units as the side of a square, from which they were able to derive two circles, the perimeter and the area circles, bearing to each other the ratio of $\pi$ to $2\sqrt{\pi}$. The nearest value to $\pi$ which they could arrive at was $\frac{22}{7}$, and the nearest values to the root of $\pi$ were two fractions ($\frac{44}{25}$ and $\frac{25}{14}$), which, multiplied together, make $\frac{22}{7}$; these they used separately as occasion required. Thus the circumferences of the perimeter and area circles are $50\times\frac{22}{7} : 100\times\frac{44}{25}$. The radii of these circles are respectively 28 and 25. On this square of 44 units the whole system of squaring the circle (or, in other words, of finding a circle whose area is equal to that of a given square) is founded.

Proceeding further, the ancients attempted to ascertain the contents of a cylinder in cubic palms by calculation, but failed to accomplish this until they had recourse to a *second unit*, or primitive inch, which enabled them to surmount the difficulty of obtaining a value for the cube-root of $\pi$, which they required when comparing a cube and cylinder of equal content.

This *second unit* was of such a dimension that it bore to the palm the ratio of 1 to $\frac{53}{18}$ $(=2\sqrt[3]{\pi})$.

By means of this they calculated that a cylinder of 6 palms, radius and height, had a content of 17,500 cubic primitive inches, and from this they arrived at the knowledge that a cylinder of four times this content—viz., 70,000 cubic primitive inches—was about $9\frac{1}{2}$ palms or 28 inches in height and radius, and was equal to a cube of 14 palms a side.

By extracting the cube-root of 70,000 they calculated the side of the double cubit cubed, in primitive inches, to be $41\frac{2}{9}$.

On this their various systems—Babylonian, Egyptian, and Akkadian—are founded, which, with variations, have spread throughout the world.

### The Primitive Inch.

The exact length of the primitive inch is recorded on the base of the Great Pyramid of Gizeh: which base is allowed to be 440 cubits of about 20·6 British inches each.

The latest measurements of this base by F. Petrie gives it as $9068\dot{8}$ British inches, which is equal to 220 double cubits of exactly $41\frac{2}{9}$ inches, or 440 cubits, of exactly $20\frac{11}{18}$ British inches. The height also is 280 cubits.

It is clear, then, that the primitive inch, the inch of the Great Pyramid, and the British inch are identical, and that the square base of the Pyramid represents the square of 440 units a side, and the height represents the radius of the perimeter circle.

It will be seen, moreover, that our linear and square measures retain the use of this unit of 22.

It is necessary first, however, to show how our yard of 36 inches has been derived from the double cubit. In order to comprehend this, it is necessary to realize that the ancients compared their systems of measures by their cubical contents, and that their weights for silver and for commercial purposes were respectively $\frac{1}{60}$ and $\frac{1}{80}$ of $\frac{1}{27}$ (double cubit cubed), this ($\frac{2}{3}$ cubit)$^3$ being the cubic foot or talent, except in Egypt, where the talent was the bushel or $\frac{1}{32}$ (double cubit cubed), also divided into 80 hon or pounds.

One cubic yard is $1{,}728 \times 27 = 46{,}656$ cubic inches.
Half a cubic yard is $864 \times 27 = 23{,}328$ „ „
Together equal to
70,000 cubic inches less $16 = \overline{69{,}984}$ „ „

that is to say, the British cubic yard is $\frac{2}{3}$ the ancient double cubit cubed less about $\frac{1}{5000}$.

Thus 36 inches correspond to $41\frac{2}{9}$ (the ancient double cubit).
,, 18 ,, ,, to $20\frac{11}{18}$ (the ancient cubit).
,, 12 ,, ,, to $13\frac{20}{27}$ (the ancient foot).

From this cubic foot we derive two pounds, one of 28·8 and one of 21·6 cubic inches.

To reduce these to grains imperial we have as a multiplier to the cubit inch—

For the year 1890, distilled water free of air, 252·245, giving 7263·2 and 5447·5 grains imperial.

For the year 1798, distilled water, 252·386, giving 7265·6 and 5449·3 grains imperial.

For the year, say, 1700, rain-water, 252·5, giving 7272·0 and 5454·0 grains imperial.

This last result is the one that is to be dealt with.

Now, it is evident, from its appearance, that this number 252·5 was not the number of grains to a cubic inch in early days, and it is apparent that while the inch has remained true the grain Troy has depreciated by 1 per cent., the original number of grains having been 250 to a cubic inch, giving 7,200 and 5,400 grains to the two pounds—numbers which exist to the present day in Europe, the latter being the Tower pound.

As confirmation of this, we have the fact that the Roman pound has continued to the present day without depreciation, and if we apply the correction of 1 per cent. we get the original weight of the Roman pound—viz., 5,184 grains.

Present weight of Roman pound = 5235
52
5,183 should be 5,184.

We are told by Lord Liverpool ('Coins of the Realm') that this Tower pound of 5,400 grains Troy was our standard pound at the Mint until the reign of Henry VIII., when it was replaced by the Troy pound of 5,760 grains Troy.

They bear to each other the ratio of 15 : 16

There is also in Europe another pound of 7,680 grains, $\frac{4}{3}$ of the Troy pound, and it is asserted by several authorities that the Saxons brought into England a 15-ounce pound, while the 7,680 grains pound is 16 ounces.

We have then the following systems:

| | British System founded on the British Yard. | | Troy System. | |
|---|---|---|---|---|
| Grains imperial | 7,200 | 5,400 | 7,680 | 5,760 |
| | 15 oz. | $11\frac{1}{4}$ oz. | 16 oz. | 12 oz. |

In order to test the relations between the pound Avoirdupois and the pounds Tower and Troy, I take them each as a fraction of the double cubit cubed, as follows:

| | |
|---|---|
| Avoirdupois pound ... ... | $\frac{1}{2560}$. |
| Troy pound ... ... ... | $\frac{1}{3037 \cdot 5}$. |
| Tower pound ... ... ... | $\frac{1}{3240}$. |

Taking the value of the double cubit cubed at 17,675,000 grains for 252·5 grains to the cubic inch (rain-water), or 250 grains to cubic inch as in early times, we get:

| | 250 Grains to Cubic Inch. | 252·2 Grains to Cubic Inch. |
|---|---|---|
| Avoirdupois pound as it is ... | 6,930 | 7,000 |
| Avoirdupois pound as it should be, $\frac{1}{2560}$ double cubit cubed | 6,834 | 6,902 |
| Tower pound $\frac{1}{3240}$ ... ... | 5,400 | 5,454 |
| Troy pound $\frac{1}{3037 \cdot 5}$ ... ... | 5,760 | 5,817 |

This shows that both the Tower and the Troy pound are 1 per cent. too light at present, while the Avoirdupois pound is 1·6 per cent. too heavy.

On the other hand, before our grain became depreciated 1 per cent. the Avoirdupois pound should have weighed 6,834 grains in lieu of 6,930. As matters stand now, taking the 5,760 grains Troy as the standard, the 7,000 grains Avoirdupois are 166 grains too heavy.

The Troy system with its Apothecaries' and Troy weight has signs of great antiquity, but it does not seem to have been used in very early times in Europe. It may have been in use in the East as far back as 700 B.C. (see weights from 'Excavations at Jerusalem,' P. E. F. Q. S., 1870, p. 330).

The pounds 80 to the cubic foot, which still remain in Europe, can all be accounted for except the Troy pound :

The Solonian pound, 4,860 grains imperial, East Europe.

The ($\frac{16}{15}$ Solonian) Roman pound, 5,184 grains imperial, Rome and Southern Italy.

The ($\frac{2}{3}$ Double cubit cubed) Tower pound, 5,400 grains imperial, Germany and Britain.

The ($\frac{16}{15}$ Tower pound) Troy pound, 5,760 grains imperial, France and Britain.

The gold coins of Europe in early days were all founded on these weights, the Aureus of 72 grains Troy and the Ducat of 54 grains Troy

representing respectively $\frac{1}{75}$ and $\frac{1}{100}$ of the Tower pound.
,, ,, $\frac{1}{72}$ and $\frac{1}{96}$ of the Roman pound.

The weight of the ducat can be traced back to early times, as it is exactly the $\frac{1}{4}$ Maccabean shekel.

The number of 250 grains to a cubic inch of rain-water was probably introduced at the time that the double cubit of 36 inches was introduced. The original numbers to each inch cube were $222\frac{2}{9}$ ($\frac{2000}{9}$) grains in Babylonia and $219\frac{3}{7}$ grains in Egypt, differing by 1·27 per cent., the number of conventional grains to a cubic inch being 220 grains.

Thus to change from ancient grains (Babylonian) to modern Troy grains we have $\frac{2000}{9} : 250 :: 8 : 9$.

$\therefore \frac{2000}{9} \times \frac{9}{8} + 1$ per cent. $= 250 + 1$ per cent. $= 252{\cdot}5$ grains to the cubic inch.

## Measures of Capacity.

Our measures of capacity appear to be of great antiquity, and are according to the system retained by the Egyptians

in Egypt, and not according to the Roman and Grecian systems, derived from Babylonia.

| | |
|---|---|
| Bushel, imperial measure | 2,218 cubic inches. |
| ,, Winchester ,, | 2,150 ,, |
| Mean - | 2,184 ,, |
| ($\frac{1}{32} \times 70{,}000$) Egyptian measure | 2,187 ,, |

### Linear Measure.

Our mile of 1,760 yards shows its origin—8 furlongs of 220 yards or double cubits each.

### Square Measure.

Our square measure also shows its origin. The unit is 10 acres of 220 yards or double cubits a side, and the next lower square dimension is the square chain of 22 yards a side.

### The Early Measures.

The early measures appear to have originated in the following sequence:

1. The binary measures, which were elaborated before the Egyptians left Babylonia, and which were retained in Egypt; the cylindric measurements for the pint, gallon, bushel, and quarter, and the hon (or pound Avoirdupois).

2. The cubical and hemispherical measures of the Babylonians and Hebrews, giving the log, bath, etc., subsequently adopted in Rome and Greece.

3. The Akkadian or Sumerian weights and measures, under which $\frac{6}{7}$ of the double cubit cubed was adopted as the unit of weight, and $\frac{1}{1000}$ part of the content of this as the mina or pound, $\frac{1}{60}$ of this being the cubic inch or shekel. This system seems to have found its way among the Pelasgi and Etruscans at a very early period (1000 B.C.), and the Roman foot and Roman pound of 5,000 grains imperial are derived from it.

4. The system of the yard of 36 inches, the foot, and the Tower pound seems to have been derived from the double cubit cubed, giving the British mile and square measure; but whether it has come from Babylonia or Egypt there seems no evidence.

5. The Solonian and late Roman pounds and the derived foot in each system appear to come from No. 4.

6. The Euboic system is evidently Egyptian.

In considering our British standards the marvel is that amid all the vicissitudes to which they have been subjected they should at the present time be able so triumphantly to testify to their ancient origin.

If we examine the weights and measures of other countries, we find individual weights and measures correct, as is the Roman pound, but there is no such collective testimony as to the ancient weights and measures as there is in our Isles.

We must feel pride in the care and supervision taken of these ancient standards at the Mint and public offices, by which these records have been handed down to us with so little deterioration and depreciation.

# THE ANCIENT CUBIT

AND

# OUR WEIGHTS AND MEASURES

## CHAPTER I

### THE ORIGIN OF THE COMMON CUBIT

Most of us pass from youth to old age without realizing that the common things of everyday life have their origin in the dim past, long before history was written. Folklore and myths and fairy tales of all countries, astronomy and the division of the circle, the signs of the Zodiac and the mapping out of the constellations of the starry firmament, writing and arithmetic, weights and measures, have all come down to us from the remotest times.

Of all common things that affect our daily life, weights and measures are amongst the most prominent, and it is proposed to pass these under consideration, and to show how they are connected with the past, and the very slight variation they have been submitted to from the earliest antiquity. It is proposed to show that all measures and weights (except the modern metrical system) have been derived from one common stock—an original unit of linear measurement, used by the Assyrians and Egyptians centuries before the Israelites became a nation.

It has been stated by Mr. Grote (Classical Museum,' 1844,

vol. i.) that *weights were determined before measures, and that measures were derived from weights,* but this is only partially true. It is correct so far that the linear measures of different nations cannot be compared directly with each other, but must be compared by their cubes—*i.e.,* cube with cube; but the original measures were linear and derived from the human body, and the original method of estimating the amounts of goods was by bulk and not by weight.

How the original linear measurements, as derived from the human body, came to have a fixed standard to endure for all time may never be discovered, although the standard itself is now known.

The cubit as derived from the human body would vary with each person, and could only be fixed by being referred in some manner to the earth's surface, or to the length of a pendulum* beating some known amount of time. Efforts have been made to show that the ancient cubit is a portion of the earth's diameter or circumference, but without success hitherto, possibly because the true length of the ancient cubit was not known. Though there is no proof that the ancients were enabled to obtain any accurate evaluation of a degree or of a minute of latitude or longitude on the earth's surface, yet we know that they did make calculations on the subject; there are statements of Livy, Aristotle, Eratosthenes, and others as to the compass of the earth, and there can be little doubt that the learned in early days may have estimated a portion of a degree of latitude on the earth's surface with some amount of accuracy, and may have approximated very closely to a minute of arc of longitude for the latitude where the estimate was made.

As a case in point, it may be mentioned that in latitude 30° N. (nearly the latitude of Southern Babylonia and of the Great Pyramid), the sixtieth part of a second of arc of longi-

* See Introduction.

tude closely approximates to the length of a common cubit or cubit of a man (*i.e.*, the fourth part of the height of a man), and the sixtieth part of a second of latitude approximates to the common cubit and a hand-breadth—*i.e.*, in the proportion of six to seven.

It will be shown that the *science of numbers* required that there should be this longer cubit of a cubit and a hand-breadth, and it seems possible that the Great Pyramid was erected at this particular point where the degrees of longitude and of latitude have the ratio of 6 : 7 to each other to record and emphasize this fact.

It is evident that the division of a second of arc into 60 parts is a very probable division* to have been made in Babylon, and the fact that the unit derived from this fraction accords with that recorded by the Great Pyramid of Egypt to $1\frac{1}{2}$ per cent. may be a mere coincidence, but it at least is worthy of thorough consideration.

The original linear measure—*the common cubit* or cubit of a man—has only come down to us incidentally, but the building cubit derived from it ($\frac{7}{6}$ common cubit) is *en évidence* in all its variety all over the world, and the records of the past bearing on it have been collected together by Mr. Flinders Petrie in a most useful volume, 'Inductive Metrology,' showing the length of the cubit deduced from most of the buildings and earthworks of importance in the world.†

These records are of the most convincing character; they show that the building cubit (speaking generally) has never been less than 16 inches, and never more than 22 inches (with the occasional exception of the cubit of 25 inches), and they show further that all cubits are derived from a common

* Because there are $360 \times 60 \times 60$ seconds in a circle, and the division of the second into 60 parts would be following the same system.

† See Introduction.

stock cubit according to fixed rules. They show that the foot, whether British, Roman, Grecian, or earlier, is but the two-thirds of a cubit, and that the inch can be traced back into remote ages before the foot came into existence.

We must consider first the records we possess concerning the origin of the common cubit.

### The Evidence of the Bible.

The references to the cubit and its subdivisions in the Bible are not very numerous.

These are: The Etzbâ, or digit (finger-breadth), only mentioned in Jer. lii. 21: 'The thickness thereof was four fingers.' Four fingers are reckoned to a hand-breadth.

The Tupah, or hand-breadth or palm. It is not known to be an anatomical word referring to the human body, and in its strict original meaning signifies 'extent'; it contains four digits or fingers (Exod. xxv. 25; 1 Kings vii. 26; 2 Chron. iv. 5; Ezek. xl. 5, xliii. 13).

The Zereth, or span, again is not strictly an anatomical word, but signifies 'expanse'; it is the extent or stretch from the end of the thumb to the end of the little finger when the fingers are extended, and this extent is equal to three palms or twelve digits (Exod. xxviii. 16; 1 Sam. xvii. 4; Isa. xl. 12; Ezek. xliii. 13).

The cubit (*ammah*, or forearm), which is supposed to be the distance measured from the elbow to the tip of the middle finger. This is the *common cubit* of 2 spans, or 6 palms, or 24 digits. References to the length of the cubit occur only in a few places in the Bible, and are very meagre.

(*a*) The cubit after the cubit of a man (Deut. iii. 11); in the measurements of Og's bedstead (Deut. iii. 11); and probably in measuring the height of Goliath of Gath (1 Sam. xvii. 4)—the common cubit.

(*b*) The cubit used in measuring the Ark and Mercy-seat,

and other small accessories of the Tabernacle and Temple—possibly also the common cubit (Exod. xxv. 10; 1 Kings vi., vii.).

(*c*) The cubit *after the first measure* (ancient) used in building the walls of the Temple, probably identical with the cubit and the hand-breadth (2 Chron. iii. 3; Ezek. xl. 5; xliii. 13).

(*d*) 'Great cubits' are also mentioned, but the passage is obscure and uncertain (Ezek. xli. 8).

There are thus practically only two cubits mentioned—the common cubit and the building cubit of a cubit and a hand-breadth or palm, and possibly a third for measuring the Temple utensils.

These terms are rendered in the Septuagint by the Greek equivalents for forearm length, span, palm, and digit, and the Talmudists also use these terms in the same manner. We may therefore safely assume that the renderings in the Authorized and Revised Versions are correct, and that they actually represented parts of the human body.

In the works of the ancient writers there are frequent notices of these measures of the body. Herodotus mentions of the Egyptians: 'An orgia is 6 feet, or 4 cubits; a foot is 4 palms and a cubit 6 palms' (ii. 149). In speaking of the walls of Babylon he says: 'The Royal cubit exceeds the common cubit by 3 finger-breadths' (iii. 78).

Among the Greeks the cubit was divided into 2 spans, or 6 hand-breadths, or 24 digits, and among the Romans into 1½ feet, or 6 palms, or 24 thumb-breadths.

It may here be mentioned that the inch has no direct connection with the human body, but is the *second, or subsidiary, unit* devised by the ancients in turning circular into rectangular measure, and has a particular connection with the palm. It is, roughly speaking, a little more than the third of a palm.

The original foot* is two-thirds of the original cubit. The

* Two-thirds of the ancient cubit cubed of rain-water gave the weight of the ancient talent.

various sizes of the foot in comparatively modern times depend on the cubits they are derived from, of which a table is given in Chapter X.

It will be found on trial that the proportions of the body used by the ancients are invariable, notwithstanding the variations of the human form—*i.e.*:

4 fingers or digits make 1 palm.
3 palms make 1 span.
6 palms (2 spans) make 1 cubit or forearm.
4 cubits make the height or stature of a man.

These relative proportions are sufficiently accurate for all ordinary purposes of measurement where extreme accuracy is not required. We have now to ascertain approximately, if we can, the height of an average fighting man, in those early days, from whom the length of the cubit may originally have been derived.

The height of the Semitic race in the towns of Europe will not serve as any guide to us, as these people have become so stunted by their surroundings and centuries of sedentary life that they are far below their natural average. The Jew of Poland (5 feet 3 inches to 5 feet 5 inches in height), whenever he gets a chance under favourable circumstances, rapidly reaches the height of 5 feet 9 inches in a generation or two (*Popular Science Monthly*, vol. liii., p. 171).

We must turn, then, to the East at the present day, and there we find that the Semitic race, living away from towns, are of goodly stature, far above the Jewish average in Europe, and many of them are tall. Many Bedouin are over 6 feet in height.

It may be suggested, then, as a shot, that the average fighting men amongst the Semitic races in the East in early days ranged in stature from 5 feet 7 inches to 5 feet 11 inches, and that for the length of the cubit the average of the taller

men would be taken—say men of 5 feet 10 inches. This shot at the truth is merely interesting in order to show how nearly the stature of a well-grown man accords with the length of the common cubit, which is known to be between 17·6 and 17·7 inches.

To put the matter in another way, we may assume that the height of fighting men in ancient times (from Egypt to Assyria) lay between 5 feet 4 inches and 6 feet 2 inches, and that the average for the cubit would be taken above the mean—say at 5 feet $10\frac{1}{2}$ inches, instead of 5 feet 9 inches. We thus arrive at extremes and a mean for the cubit and its parts as follow:

| | | | | | |
|---|---|---|---|---|---|
| Cubit, from 16 to $18\frac{1}{2}$ inches. | Higher | average, | | 17·65 | inches. |
| Span, from 8 to $9\frac{1}{4}$ | „ | „ | „ | 8·825 | „ |
| Palm, from $2\frac{3}{4}$ to $3\frac{1}{12}$ | „ | „ | „ | 2·941 | „ |
| Digit, from $\frac{11}{16}$ to $\frac{37}{48}$ | „ | „ | „ | ·735 | „ |

With these average values of the cubit we may consider the nature of the first measures used by men when they commenced to barter and sell.

Their first notion would be to sell by bulk—so many heaps or baskets full of corn and so many pitchers or pots of oil and wine. Corn would probably be the first article sold by it (be it barley, millet, rye, or wheat); for this they would have baskets of primitive shapes—truncated cones, hemispheres, and, for accuracy, cylinders.

The cylinder of one cubit height and radius would give a measure of about 500 lb. weight of corn, which could be moved by hand, and the cylinder of half a cubit height and radius would give a measure of about 60 lb. weight of corn, which would be about a man's load. For smaller measures there would be the cylinder of a quarter and another of an eighth of a cubit in height and radius, each measure having one-eighth the capacity of the next larger in size. We thus

arrive at four cylindrical measures, just as we have in our British measures, viz., quarters, bushels, gallons, and pints, and on the same system, each one eight times the capacity of the smaller one. It will only be necessary to state the capacity of the quarter in each case to show how closely they coincide:

| Ancient Assumed Cylindrical Measure of 17·7 Inches Height and Radius. Cubic Inches. | Imperial Measure. Cubic Inches. | Winchester Measure. Cubic Inches. |
|---|---|---|
| 17,273 | 17,745 | 17,201 |

It seems plain that our British measure of capacity, the quarter, accords closely with the cylinder, based on the original cubit of 6 palms derived from a man of 5 feet 10½ inches height.

Mr. H. J. Chaney, in 'Our Weights and Measures,' states that the British standard dry measures are cylinders, with their diameters double their depth; and thus the British standard bushel at the present day, as preserved in the Standards Office, Westminster, is a cylinder of height equal to radius and each half a cubit of 17·80927 British inches.

It may be objected that our British weights and measures have been frequently altered and readjusted from time to time. This certainly is the case, but these readjustments have been continually bringing back our measures to their original standard. As an indication of this, Mr. Flinders Petrie ('Inductive Metrology,' p. 107) gives sixteen different cases of old buildings from the twelfth to the fifteenth century, in which he shows that the inch has not varied more than from ·984 to 1·013 the modern inch, and that the mean is ·9998 inch, and considers that our inch, as now in use, has not varied an appreciable amount, on an average, *for centuries.*

Again, Mr. Chaney states: 'There appears to be no doubt that the present Imperial standards have been accurately derived from those of Queen Elizabeth, and that these latter

were derived from those of King Henry VII.' He further states that the standard yard of 36 inches (A.D. 1496) still exists, and is probably of the same length as the *Old Saxon yard* (*see* 'Descriptive List of Ancient Standards,' Papers presented to Parliament by command, C. 6541, 1892).

As an interesting test, we may take the more ancient building cubit given by Mr. Petrie from various parts of the world, and taking six parts out of seven, arrive at the length of the common cubit:

| | | |
|---|---|---|
| Assyrian and Babylonian ... | 21·30 | 18·29 |
| | 19·97 | 17·$\dot{1}$ |
| | 20·61 | 17·$\dot{6}$ |
| Greece, double Pythic foot ... | 19·30 | 16·533 |
| Prehistoric remains, Britain ... | 21·39 | 18·30 |
| Prehistoric remains, France ... | 21·89 | 18·76 |
| Royal Egyptian ... ... ... | 20·77 | 17·802 |
| | 20·61 | 17·$\dot{6}$ |

We thus see that the average early cubit of 6 palms ranged between 17 and 18·76 inches, and that the cubit as derived from our British dry-measure standard closely accords with the average—*i.e.*, closely accords with the cubit of Egypt. The derivation of this cubit is considered in Chapter IV.

The next question to discuss is a larger measure for corn, and a smaller measure than the pint for oil, etc.; also the turning of circular measure into rectangular measure.

By trial it would be found that four cylinders of 6 palms would just fill one cylinder of $9\frac{1}{2}$ palms:

$$(9\tfrac{1}{2})^3 = 857\tfrac{3}{8} : (6)^3 = 216 :: 4 : 1 \text{ nearly.}$$
$$\text{Discrepancy, } 6\tfrac{5}{8} \text{ in } 857.$$

Again, it would be found that a cylinder of $9\frac{1}{2}$ palms is nearly equal to a cube of 14 palms a side:

$$(9\tfrac{1}{2})^3 \times \frac{60}{19} = 2{,}707\tfrac{1}{2};\ (14)^3 = 2{,}744.$$

They would thus arrive at two measures of equal capacity,

a cylinder and a cube, each about four times the capacity of our British quarter, and the side of the cube is the double cubit of 7 palms.

Thus they arrived at the standard measure of the East, the double cubit cubed or cylinder of $9\frac{1}{2}$ palms, of which the modern equivalent is the chaldron or four quarters.

At the lower end of the scale they would find no ready reduction of the pint (a cylinder of $\frac{3}{4}$ palm radius and height), but they would find that a palm cubed would equal about $1\frac{1}{4}$ pints, or $\frac{1}{10}$ gallon. Actually, $10\frac{3}{4}$ cubic palms go to a cylinder of $1\frac{1}{2}$ palms (gallon).

They would thus find by calculation and by practical tests:

Cube of 14 palms a side $(14)^3 = 2{,}744$ cubic palms.

Cylinder, height and radius $9\frac{1}{2}$ palms $(9\frac{1}{2}*)^3 \times \frac{60}{19}$ = about $2{,}707\frac{1}{2}$ cubic palms.

Chaldron or 4 quarters $= 4 \times 8 \times 8 \times 10\frac{3}{4} = 2{,}740$ cubic palms.

The use of $10\frac{3}{4}$ would be inconvenient for practical purposes, and they would probably take 10 instead. Taking, therefore, 10 instead of $10\frac{3}{4}$, they would have $4 \times 8 \times 8 \times 10 = 2{,}560$, and the measures would run thus, as they do to the present day:

4 quarters, 2,560 cubic palms nearly.
1 quarter (= 8 bushels), 640 cubic palms nearly.
1 bushel (= 8 gallons), 80 cubic palms nearly.
1 gallon (= 10 cubes), 10 cubic palms nearly.

If we apply practical tests to these measures, we shall find that the difference of the sides of a cube of $\frac{1}{10}$ gallon and a cube of $\frac{1}{10\frac{3}{4}}$ gallon is too small for measurement, lying between about 3 and 2·94 inches; the difference in content is also too small, being only about $\frac{1}{5}$ cubic inch.

* For the approximation to $\pi$ see p. 16.

If we compare the higher measures, viz.: the cylinder of $9\frac{1}{2}$ palms with a cube of 14 palms, there certainly should be a difference of nearly 200 cubic palms, or 8 per cent., in capacity; but it is difficult to make these measures rigidly accurate even at the present day, and the difference would not have been appreciated in very early times. Subsequently, no doubt, it was rectified, as will be seen in succeeding chapters.

### Computation by Grains.

Although in later times the computation of cubic content is known to have been made by means of the weight of rain-water, yet it seems probable that the original computation would have arisen from dry measure and from grains of corn —probably of barley.

Probably of barley, because all early tradition—British, Jewish, etc.—seems to hinge upon the weight, length, breadth, etc., of barley grains. According to Dr. Birch ('Egypt: Ancient History from the Monuments'), wheat did not come into use in Egypt till after the Fourth Dynasty, but barley was in use in very early times.

It is not proposed at this point to do more than indicate the method by which the weight and bulk of grains were first connected with weights and measures. It was found that a cubic palm would just hold 4,000 (say, 4,009) ancient grains of barley, so that $2{,}744 \times 4{,}009$ gave 11,000,000 grains of barley to a cube of 14 palms.

The relative weight of a measure of barley (pressed down) compared with a measure of rain water was as 5 to 7 (as will be shown hereafter), so that a cube of 14 palms would hold in water the weight of 15,400,000 ancient grains.

When the weight by water was once brought into use, it would be considered more reliable, as it is not subject to such fluctuations in weight as grain is; but for the commencement

of measure by weight, the standard of 4,000 grains to a cubic palm was probably adopted.

TABLE I.

*Values of Original Measures.*

| Measure. | Calculation.* | Cubic Palms. | |
|---|---|---|---|
| | | 4 Quarters. | Corrected to $\pi$. |
| Cube, 14 palms a side ... | — | 2,744 | 2,744 |
| Cylinder, $9\frac{1}{2}$ palms radius and height | $(9\frac{1}{2})^3 \times \frac{34}{19} \times \frac{34}{19}$ | $2,745\frac{1}{2}$ | $2,693\frac{1}{4}$ |
| Box, $17 \times 17 \times 9\frac{1}{2}$ ... | — | $2,745\frac{1}{2}$ | $2,745\frac{1}{2}$ |
| Cylinder, 6 palms radius and height | $6^3 \times \frac{19}{6}$ | $\frac{1}{4} \times 2,736$ | 2,713 |

* For the calculations see pp. 15, 16.

## CHAPTER II

### EARLY TRIALS TO SQUARE THE CIRCLE WITH THE COMMON CUBIT OF SIX PALMS

The relations of a circle to a square must have attracted very early attention, and the area of a circle in square measure would probably have been approximated to with some accuracy long before the result could be expressed by symbols.

It is easy for an average workman, even with primitive instruments, to strike out a circle from some stuff of uniform weight and thickness, as, for example, a hide or metal plate, and to balance its weight against a piece of similar stuff cut square. At a very early period this, if carefully done, would have given fairly accurately the relation between a circle and a square of the same area—between the diameter of one and the side of the other; and also by repeated trials it could have been ascertained that the circumference of a circle is over three times the length of the diameter.

De Morgan relates in 'A Budget of Paradoxes' that two values of $\pi$ were obtained by actual measurement by artisans—3·125 and 3·1406. The second result was obtained by a joiner in 1863 by means of a disc, of 12 inches diameter, rolling upon a straight rail. The mechanics in early days could have worked quite as accurately, and as the value of $\pi$ thus obtained is as much under as the values given by

Archimedes are both under and over, viz., $\frac{22}{7}$ and $\frac{223}{71}$, we may be quite sure that the ancients were able to make a very close approximation to the truth by practical trials.

In a similar manner there would be no difficulty in obtaining the relative capacities of cylinders, cubes, pyramids, and cones.

It is probable that the ancients would have little difficulty in their practical tests in comparing the contents of cylinders and cubes, and that they would arrive within 10 per cent. of the truth without a great number of trials. Their great difficulty would be in expressing their dimensions accurately in palms and digits without the use of decimals. They could work with fractions, but in a cumbrous manner.

It is assumed that they first arrived at their results by actual measurements, and not by any such mathematical tests as those used by Archimedes—the inner and outer polygon. The geometry of the Egyptians was apparently confined to the relation of numbers to each other in connection with areas and cubes and practical tests.

Necessarily, the first problem would be the expression of the circumference of a circle in terms of the diameter, and the obtaining a circle whose area is equal to that of a given square, without having recourse to unwieldy fractions. In other words, they required to *square the circle.*

A diameter of 6 palms (or a common cubit) would be the natural starting-point for their trials, and as they knew from rough measurements that the circumference is about three times the diameter, they would arrive at once at 19 as a possible ratio to 6 (circumference to diameter $\frac{19}{6} = 3{\cdot}1\dot{6}$). That this number 6 was the original diameter used by them may be considered probable, because in the Hieratic Papyrus (1600 B.C.) of the Rhind collection it is stated as one of the geometrical problems 'to find the surface

of a circular area whose diameter is 6 units' (Allman's 'Greek Geometry,' p. 16).

In another portion of this papyrus it is stated that a circle can be squared by taking eight parts of a diameter of 9 parts, and erecting a square on the eight.

This is equivalent to using $\left(\frac{4}{3}\right)^4$ as $\pi\left(\left(\frac{4}{3}\right)^4 = 3{\cdot}1604\right)$: a very convenient value for dealing with, as it has a square root* of whole numbers $\left(\frac{4}{3}\right)^2$. This gives a circumference of $28\frac{4}{9}$ and area 64. $2\pi r = 9\left(\frac{4}{3}\right)^4 = 28\frac{4}{9}$ and area $8^2 = 64$.

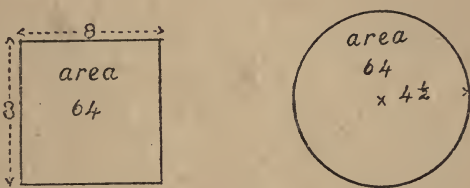


FIG. 1.

This approximation to $\pi$ is very close to that of $\frac{19}{6}$ already given with a diameter of 6, the difference between $\left(\frac{4}{3}\right)^4$ and $\frac{19}{6}$ being only $\frac{1}{162}$; *e.g.*, $\left(\frac{512}{162} \text{ and } \frac{513}{162}\right)$.

If we now compare the area of a circle with diameter of 6 with that of square of equal area, taking $\pi = \frac{19}{6}$ and $\sqrt{\pi} = \left(\frac{4}{3}\right)^2$, we get area of square $(r\sqrt{\pi})^2 = \left\{{}^{3}\left(\frac{4}{3}\right)^{2}\right\} = 28\frac{4}{9}$ and area of circle $\pi r^2 = \frac{19}{6}\left(\frac{6}{2}\right)^2 = 28\frac{1}{2}$.

* Which they required for side of square ; see p. 17.

The areas do not quite agree because $\frac{19}{6}$ has no square root which can be expressed in manageable fractions.

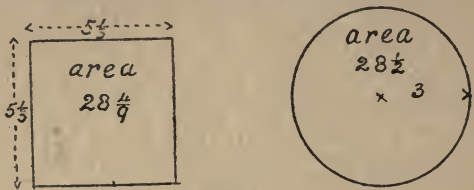


FIG. 2.

Taking other values for radius, and a slightly different ratio of the circumference and the radius (as must be done when whole numbers or manageable fractions are to be adhered to), a variety of circles would be squared. Probably the next step would be to reverse $\frac{19}{6}$ as a fractional value of $\pi$, and take $\frac{60}{19}$ in its place and in lieu of it. It differs from it as 361 : 360. Here, again, we may use $\frac{16}{9}$ as the square root, or, better still, $\frac{34}{19}$. They compare as follows:

$$\frac{19}{6}=\frac{9747}{6\times19\times27}. \qquad \frac{16}{9}=\frac{304}{9\times19}.$$

$$\left(\frac{4}{3}\right)^4=\frac{9725}{6\times19\times27}. \qquad \frac{34^*}{19}=\frac{306}{9\times19}.$$

$$\frac{60}{19}=\frac{9720}{6\times19\times27}. \qquad \left(\frac{34}{19}\times\frac{30}{17}\right)=\frac{60}{19}.$$

Now, taking a circle with a radius of $\frac{19}{6}$, we have for its area $\frac{60}{19}\times\frac{19}{6}\times\frac{19}{6}=\frac{95}{3}=31\frac{2}{3}$, and for the square: area $=(\text{side})^2$ $=(\sqrt{\pi}r)^2=\left(\frac{34}{19}\times\frac{19}{6}\right)^2=(5\frac{2}{3})^2=32\frac{1}{9}$.

* For the fractions $\frac{34}{19}$ and $\frac{30}{17}$ see p. 18.

It is to be noted that $\frac{60}{19} \times \frac{19}{6} = 10$, and their ratio is as 360 : 361, so that they were probably accepted as equivalent quantities.

Supposing they were taken as two adjacent sides of a square, the area would be 10, and a circle of similar area would $= \frac{19}{6} \times \frac{34}{19} \times \frac{30}{17} = 10$.

The Hindoo writer Brahmagupta (about 650 A.D.), in his attempt to rectify the circle, gives as his result $\sqrt{10} = \pi$, and the origin of his geometry is ascribed to the works of Hero of Alexandria (125 B.C.), so that this view as to the square of $\pi$ equalling 10 may have come from the Egyptians.

They probably now found out that by taking a square of (a multiple of 6) 30 units a side two circles can be formed: one whose perimeter is equal to the perimeter of the square, and the other whose area is equal to the area of the square, and that the radius of the perimeter circle is 19, and of the area circle 17, or as $\pi : \frac{1}{2}\sqrt{\pi}$. In this latter case we must take $\frac{53}{17}$ as the approximation to $\pi$.

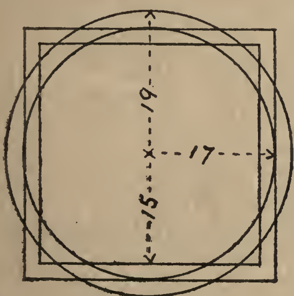


FIG. 3.

$(19)^2 \times \frac{60}{19} = 1{,}140$ area of perimeter circle.

Circumference $2 \times \frac{60}{19} \times 19 = 120$, or $4 \times 30$.

$(17)^2 \times \frac{53}{17} = 901$ area of area circle.

$(30)^2 \qquad = 900$ area of square.

For $2\pi r = 120$, $r = \frac{120}{2\pi} = \frac{60}{\pi} = \frac{60}{\frac{60}{19}} = 19$.

$$\text{For } \pi r^2 = 900,\ r^2 = \frac{900}{\pi} = \frac{30}{\sqrt{\pi}} = \frac{30}{\frac{30}{17}} = 17.$$

This discovery seems to have been locked up in Egypt (in the Great Pyramid), for there is no record of its being known to the Greeks or Romans.

In attempting to square these circles, the Egyptians found that they required the square root of $\pi$, as well as the value of $\pi$ itself,* and they must have it in whole numbers, or fractions they could deal with, otherwise the area of the square and of the circles could not all come out in numbers they could manage. They found that the use of these two circles solved their difficulties, for if certain functions of these circles (viz., the circumferences, radii, sides of squares, and areas) are put down in order so that those of the area circle are uppermost, it will be found that the fractions formed respectively by the radii and sides when the numerators are multiplied by 2 give approximate square roots which when multiplied together make the value of $\pi$ used.

EXAMPLE.

| | Circumference. | Radius. | Side. | Area. |
|---|---|---|---|---|
| Area circle ... | — | 17 | 30 | 900 |
| Perimeter circle ... | 120 | 19 | 34 | 1,156 |

By which we obtain from radii, $\frac{34}{19}$ ($=1{\cdot}789$) for $\sqrt{\pi}$.

And from sides, $\frac{30}{17}$ ($=1{\cdot}764$) for $\sqrt{\pi}$.

$$\frac{34}{19} \times \frac{30}{17} = \frac{60}{19} = (3{\cdot}15+) \text{ for value of } \pi.$$

These are, of course, not the true square roots of $\frac{60}{19}$, any

* To enable them to obtain a convenient value from the side of square of equal area from formula $s = r\sqrt{\pi}$.

more than $\frac{60}{19}$ is the true value of $\pi$; but the ancients used them as the true roots, and used them very skilfully, so as to arrive at results, in turning square into circular measure, closely approximating to the truth. For the sake of brevity, I call these roots, when referring to them, skew roots. That belonging to the area circle, $\sqrt{\pi c}$; that belonging to the perimeter circle, $\sqrt{\pi b}$; and the approximate value of $\pi = \pi a = \sqrt{\pi b} \times \sqrt{\pi c}$.

$$\text{So that } \left(\frac{30}{17}\right)^2 = \pi c.$$

$$\left(\frac{34}{19}\right)^2 = \pi b.$$

$$\frac{60}{19} = \left(\frac{30}{17} \times \frac{34}{19}\right) = \pi a.$$

These approximations may at the present day seem very wide of the truth, but it is to be recollected that an error of 6 per cent. in a content or bulk is practically reduced to an error of about 2 per cent. when evenly distributed over the sides, and in any ordinary measure such an error cannot be detected. If the values taken be tested with the old measures of capacity in our Standard Offices, it will be found that this simple, ready method of the ancients was nearly as accurate in practical results as our own.

It may now be assumed that the ancients tried other circles and combinations in addition to those above enumerated, and in order to show what they had available for use, the following list of radii is given up to 9:

| Circumference. | Radius. | Area. | |
|---|---|---|---|
| $\frac{19}{6} \times 2 \times 3 = 19$ | 3 | $\frac{19}{6} \times 9 = 28{\cdot}5.$ | $\pi = 3{\cdot}1\dot{6}.$ |
| $\frac{25}{8} \times 2 \times 4 = 25$ | 4 | $\frac{25}{8} \times 16 = 50.$ | $\pi = 3{\cdot}125.$ |
| $\frac{16}{5} \times 2 \times 5 = 32$ | 5 | $\frac{16}{5} \times 25 = 80.$ | $\pi = 3{\cdot}20.$ |

| Circumference. | Radius. | Area. | |
|---|---|---|---|
| $\frac{19}{6} \times 2 \times 6 = 38$ | 6 | $\frac{19}{6} \times 36 = 114.$ | $\pi = 3 \cdot 1\dot{6}.$ |
| $\frac{22}{7} \times 2 \times 7 = 44$ | 7 | $\frac{22}{7} \times 49 = 154.$ | $\pi = 3 \cdot 142 + .$ |
| $\frac{25}{8} \times 2 \times 8 = 50$ | 8 | $\frac{25}{8} \times 64 = 200.$ | $\pi = 3 \cdot 12.$ |
| $\frac{19}{6} \times 2 \times 9 = 57$ | 9 | $\frac{19}{6} \times 81 = 256 \cdot 5.$ | $\pi = 3 \cdot 1\dot{6}.$ |

These circles are calculated with $\pi = \frac{3r+1}{r}$, thus ensuring a whole number for circumference and also for area, except in two instances where there is a fraction of $\frac{1}{2}$.

Of these, that approaching most nearly to $\pi$ is $\frac{22}{7}$, and if we examine all possible simple fractions with denominators ranging up to 100, we shall find only one $\left(\frac{223}{71}\right)$ which approximates more nearly to true $\pi$, and that only in a very minute degree. There is one other which runs $\frac{22}{7}$ very closely, viz., $\frac{157}{50}$.

| | | Differ: with true $\pi$. |
|---|---|---|
| $\frac{22}{7}$ | $= 3 \cdot 14285$ | $+ \cdot 00126$ |
| $\frac{223}{71}$ | $= 3 \cdot 14084$ | $- \cdot 00075$ |
| $\frac{157}{50}$ | $= 3 \cdot 14$ | $- \cdot 00159$ |
| $\pi$ | $= 3 \cdot 14159$ | |
| $\frac{355}{113}$ | $= 3 \cdot 14159$ | |

We may therefore look upon $\frac{22}{7}$ as the nearest approach to $\pi$

that can be obtained with a fraction with a low denominator, and no doubt the ancients gradually arrived at the same conclusion.

### THE TWO CIRCLES DERIVED FROM A SQUARE OF 44 A SIDE GIVE THE BEST APPROXIMATION TO $\pi$ AND $2\sqrt{\pi}$ FOR CALCULATING AREAS.

As yet we have used rather inefficient values of $\pi$. To enable us to use the closest and most convenient approximation of $\pi$, $\left(\text{viz., } \frac{22}{7}\right)$ to calculate side of square from radius $(s = r\sqrt{\pi})$, we must know its square root.

But $\frac{22}{7}$ has no square root represented by a manageable fraction. None of the fractions with denominators ranging up to 100, that approximate to $\pi$, do possess such square roots, except that used by Ahmes, $\frac{256}{81} = \left(\frac{4}{3}\right)^4$.

We desire, then, to obtain such a fractional value for $\sqrt{\pi} = \sqrt{\frac{22}{7}}$ as will give whole numbers or numbers consisting of whole numbers and convenient fractions for the other functions of the unit obtained by its means. We must then resort to skew roots of the form $\sqrt{\pi b}$ and $\sqrt{\pi c}$ as explained above (p. 18).

It was discovered that a square of 44 a side, giving area circle of radius 25 and perimeter circle of radius 28, would by the method shown on p. 18 give the skew roots we require, viz., $\frac{44}{25}$ for $\sqrt{\pi c}$ and $\frac{25}{14}$ for $\sqrt{\pi b}$, the product of these fractions being $\frac{22}{7} = \pi a$.

The table which follows compares the functions of the circles obtained by using the true value of $\pi$, and those

obtained by this method show how close is the approximation.

| Circumference. | Radius. | Area. | Side. |
|---|---|---|---|
| Area circle $\pi$ 50 ... ... | 25·0 | 1,963·49 | 44·3 |
| Perimeter circle, $\sqrt{\pi}$ 100 ... | 28·2 + | 2,500·0 | 50 |
| Area circle $\frac{22}{7} \times 50$ ... ... | 25·0 | 1,936 | 44 |
| Perimeter circle $1{\cdot}76 \times 100$ ... | 28·0 | 2,500 | 50 |

Fraction used in obtaining side of square in area circle is $\frac{44}{25}$ $(=1{\cdot}76)$ or $\sqrt{\pi c}$.

Fraction used in obtaining side of square in perimeter circle is $\frac{25}{14}$ $(=1{\cdot}78+)$ or $\sqrt{\pi b}$.

$$\frac{25}{14} \times \frac{44}{25} = \frac{22}{7} = 3{\cdot}1428+$$


FIG. 4.

This method of turning square into circular measure approximates more closely to the truth than any other under which whole numbers are required for sides, areas and radii, and it is proposed that the use of the numbers 22, 25 and 28 in early times originated in this manner. For example, the use of 440 cubits as the base of the Great Pyramid.

It will be seen that this method of expressing the functions of circles will be used further on, on p. 24, for arriving at the measures of capacity and deducing the *second unit* or primitive inch.

### Measures of Capacity.

#### *The Second Unit.*

So far, by the artifice of using factors or skew roots for the approximate value of $\sqrt{\pi}$ it is possible to turn square measure into circular measure, and *vice versâ,* with some degree of accuracy, keeping to whole numbers, but by no possibility can whole numbers be adhered to in turning rectangular measure into cylindrical measure without introducing a second or separate unit for the circular measure.

For example, whatever value may be assigned to the radius of a cylinder (of equal height and radius), the cube equal in content to that cylinder must have a side equal to the radius multiplied by the cube root of the value assigned to $\pi a$.

The ratio of these two units, therefore, for any cube and the derived cylinder,* will be as $1 : \sqrt[3]{\pi b}$ or some multiple of it, where $\pi b$ is the particular approximation to $\pi$ that has been used.

* Content of cylinder $= \pi r^3$.
Content of cube $= s^3$.
$\therefore \quad \pi r^3 = s^3$.
$\therefore r \sqrt[3]{\pi} = s$.

The original unit in use was a palm, and it is necessary for easy reference to give the derived unit some denomination. This I propose to call the *primitive inch,* because no doubt it is the origin of the inch used in all measures of capacity at an early period, but probably only used as a unit of linear measure when the foot came into use in Greece in lieu of the cubit.

In order to find the value of the ratio of the palm to the primitive inch we must find the cube root of the value assigned to $\pi$, which in the case of 14 palms a side is $\left(\frac{25}{14}\times\frac{25}{14}\right)$. It seems a somewhat tough job to arrive at this without logarithm books and without an efficient decimal notation, but it can be solved by trial and error as follows:

After many trials let it be assumed that $\frac{53}{36}$ is the cube root desired; we have to show that

$$\left(\frac{53}{36}\right)^3 = \frac{67}{21} = \left(\frac{25}{14}\right)^2$$

$$(53)^3 = 148{,}877 \text{ : just 11 short of } 148{,}888 = \frac{2}{9}\times 670{,}000$$

$$(36)^3 = 46{,}656 \text{ : just 10 short of } 46{,}666 = \frac{2}{9}\times 210{,}000$$

$$\therefore \left(\frac{53}{36}\right)^3 = \frac{67}{21} \text{ nearly.} \qquad \frac{67}{21} = 3{\cdot}1904$$

$$\left(\frac{53}{36}\right)^3 = 3{\cdot}1902$$

$$\text{difference} \quad 0{\cdot}0002$$

We have now to show that $\frac{67}{21} = \left(\frac{25}{14}\right)^2$

$$\frac{67}{21} = \frac{67}{3\times 7} = \frac{67\times 28}{3\times 14\times 14} = \frac{1876}{3\times 14\times 14}$$

$$= \text{nearly } \frac{1875}{3\times 14\times 14} = \frac{3\times 25\times 25}{3\times 14\times 14} = \left(\frac{25}{14}\right)^2$$

$$\left(\frac{25}{14}\right)^2 = 3\cdot1887$$
$$\frac{67}{21} = 3\cdot1904$$
$$\text{difference} \quad 0\cdot0017$$

$$\therefore \left(\frac{53}{36}\right)^3 = \left(\frac{25}{14}\right)^2 \text{nearly.} \qquad \left(\frac{53}{36}\right)^3 = 3\cdot1902$$
$$\left(\frac{25}{14}\right)^2 = 3\cdot1887$$
$$\text{difference} \quad 0\cdot0015$$

Therefore $\frac{53}{36}$ approximates to $\sqrt[3]{\pi b}$ when $\pi b = \left(\frac{25}{14}\right)^2$. And the ratio of the palm to the primitive inch is a multiple of $\frac{53}{36}$, *i.e.*, of 1·472. (True value of $\sqrt[3]{\pi} = 1\cdot4646$.

The smallest multiple of this cube root $\left(\frac{53}{36}\right)$ that can conveniently be used when the area and perimeter circles have $\frac{22}{7} \times 50$ and $1\cdot76 \times 100$ for circumference respectively is 2, because then the radius of the first circle is 25 and the area of the second circle is 2,500.

The ratio, therefore, of the palm to the primitive inch is as $\frac{53}{18} : 1$.

We have already shown that the palm, as derived from the human frame, measures from 2·6 inches to 3·08 inches. We therefore now become aware that the primitive inch must correspond closely to the British inch.

---

We are now in a position to ascertain the content in primitive inches of a cylinder equal to a cube of 14 palms a

side, the cylinder being $9\frac{1}{2}$ palms or 28 primitive inches in radius and height:

Cylinder radius and height 28 primitive inches =

$$r^3\pi = (28)^3 \times \left(\frac{25}{14}\right)^2 = 70{,}000 \text{ cubic primitive inches.}$$

Rectangular box, base* $(50)^2 \times$ height $28 = 70{,}000$ cubic primitive inches.

We now require to find the value of the side of the cube of 14 palms in primitive inches:

$$14 \times \frac{53}{18}\dagger = \frac{371}{9} = 41\tfrac{2}{9} = \text{double building cubit.}$$

$$20\tfrac{11}{18} = \text{building cubit.}$$

*Note.*—70,000 seems to be a likely number to be adopted for the units in a standard measure, as there was a tendency to use multiples of 7 in early days. There are numerous cases of this in the Old Testament. Mr. F. Petrie ('Pyramids of Gizeh,' p. 83) states that the stones in the Pyramid average $(50)^2 \times 28 = 70{,}000$ cubic inches each.

---

In Chapters I. and II. I have attempted to show that the ancients, while basing their computations on the *sexagesimal system*, were forced by the science of numbers to substitute a cubit of 7 palms for one of 6 palms in measurements connected with *squaring the circle* and reducing cylindrical measurements to rectangular measurements, thus leading to the use of multiples of 5, 7 and 11 in connection with the standards of measure, the convenience of which is seen in the measures of Eurasia and Africa to the present day.

* Base of this box is equal in area to perimeter circle, see p. 18.

† $\frac{53}{18}$ is the lowest possible ratio of palm and inch.

# CHAPTER III

## MATHEMATICS OF THE ANCIENTS

Greek tradition assigned the special development of geometry to the Egyptians, and that of the *science of numbers* either to the Egyptians or the Phœnicians.

Thales, about 600 B.C., founded at Miletus the earliest Greek School of Mathematics. He was said to have been a Phœnician, and he studied astronomy and mathematics in Egypt.

Pythagoras, a contemporary of Thales, studied mathematics for several years at Thebes (Egypt) before starting his lectures at Samos and in Southern Italy.

These early Greek schools taught that the theory of the universe was to be found in the *science of numbers*, and Pythagoras is said to have discovered that the tone sounded by a vibrating string depended (other things being the same) only on the length of the string, and, in particular, that the lengths which give a tone, its fifth and octave were in the ratio $1 : \frac{2}{3} : \frac{1}{2}$.

Two of the problems in which these schools were most particularly interested were : (1) the duplication of a cube—that is, the determination of the side of a cube whose volume is double of that of a given cube ; (2) the squaring of a circle—that is, the determination of a square whose area is that of a given circle.

About 300 B.C. Eudoxus gave a proof that the volume of a pyramid is one-third of that of a prism or cylinder on the same base, and of the same altitude (Euc. xii. 7, 10).

In 332 B.C. Egypt was conquered by Alexander the Great and became a Greek kingdom under the Ptolemies. Then arose the University of Alexandria (300 B.C. to 641 A.D.), which flourished under Greek, and, subsequently, under Roman dominion. The first director of the mathematical department of this University was Euclid—a Greek by descent, probably educated at Athens. He compiled a work (Euclid) from the works of previous writers, and died 275 B.C.

Archimedes the Syracusan studied in the University of Alexandria, and returned to Sicily when he had completed his studies; he died 212 B.C. He gave a proof that the volume of a sphere was equal to two-thirds of the circumscribing cylinder. He also, in measuring the circumference of a circle in regard to the diameter, showed that $\pi$ is less than $\frac{22}{7}$ and greater than $\frac{223}{71}$. He also adopted a system of numeration based on a decimal system, and it is conjectured that he had some symbolism based on this system which enabled him to make his investigations with facility.

It is not asserted that Thales, Pythagoras, Euclid, and Archimedes discovered the solution of the problems connected with their names, but, rather, that they gave intelligible proofs which could be understood by the students in the schools; and there seems every reason for supposing that the mathematical knowledge of the Greek schools was derived from the Egyptians, Phœnicians and Assyrians, who had made great advance in astronomy, geometry, and building, during centuries anterior to the rise of Greek civilization.

There seems to be no evidence that the Greek schools made any progress in the practical application of the squaring of the circle—*i.e.*, they do not appear to have attempted to

prove that a cube of a certain length of side was equal in content to a cylinder of given height and radius, although they must have been familiar with the idea in connection with their weights and measures. Perhaps this had already been done for them practically by the Babylonians and the Egyptians, from whom they derived their measure, and possibly they were incapable of giving rigid proofs of that which they knew the truth of by practical tests.

Strabo tells us that geometry was introduced into Greece from Egypt, and astronomy and arithmetic from Phœnicia, and it is evident that we must look to the Egyptians and Phœnicians for the origin of the system of weights and measures introduced into Greece, and for the system adopted in turning circular measure into square measure.

Fortunately, a papyrus written by an Egyptian priest named Ahmes, between the years 1700 and 1100 B.C., exists in the Rhind Collection of the British Museum, treating upon the arithmetic of the Egyptians of that period. The work consists of a collection of problems in arithmetic and geometry, and the process by which the results are obtained can be gathered. The first part deals with the reduction of fractions of the form $2/(2n+1)$ to a sum of fractions whose numerators are unity; and it is to be observed that this method is adopted so late as the time of Archimedes: *e.g.*, in his celebrated problem about the bulls and cows belonging to the sun he writes $\frac{1}{7}+\frac{1}{6}$ instead of $\frac{13}{42}$.

This was done probably for facility of computation in adding and subtracting by means of tables.

So late as A.D. 944 we are told in the 'History of Mathematics' that when a certain mathematician (an expert in the science) wanted to divide, for example, 6152 by 15, he had to try all the multiples of 15 until he arrived at 6000; this gave him a quotient of 400 and a remainder of 152; he then began

again with the multiples of 15 until he arrived at 150, with a quotient of 10 and a remainder of 2. Hence he arrived at the quotient of 410 with remainder of 2.

The difficulty of multiplying and dividing, without a decimal notation, was so great that the ancients probably kept tables, or 'ready-reckoners' for use in calculating. In dealing with fractions, they had to resort to devices and artifices, and could, as a rule, only arrive at approximations.

For extracting the square and cube roots of quantities no doubt they had special tables. For the ordinary purposes of life they probably (in the civilized parts of Asia, of Europe, and Africa) used the instrument called *abacus*, or *swanpan*, by which they were enabled to add and subtract rapidly.

## The Progress of the Ancients.

It appears to be generally recognised that Babylonia, extending from 30° to 33° N. Lat., is the cradle of the early civilization of Asia, Europe and Egypt.

From this centre mankind seems to have gone forth with their knowledge of the length of the cubit, of cylindrical measures, of the division of the circle, of the day and of the year.

Here the degree of latitude bears to the degree of longitude the ratio of about 7 : 6, and it was here that by the science of *numbers* the ancients were forced to develop their common cubit of 6 spans into a double cubit of 7 spans in order effectively to *square the circle* and reduce cylindrical measure into rectangular measure.

It was probably here that the ancients came to the conclusion that a minute of arc of the meridian measured eight times 220 double cubits of 7 palms, and thus fixed permanently the length of the cubit, the number 220 representing

the length of side of a square giving the two circles from which the approximate values of $\pi$ and $\sqrt{\pi}$ were derived.

So far the system of weights and measures of the Babylonians and Egyptians appear to have been on all fours, *i.e.*, no separation of these people had taken place.

After this, however, the Egyptians appear to have separately developed their binary system (Chapter V.), and the Babylonians appear to have developed two systems, one adopted by the Semitic races (Chapter VI.), and the other by the Sumerians and Akkadians (Chapter XI.).

## Mathematics of the Egyptians.

There was no difficulty in reckoning high numbers, and they possessed, for integers, a convenient decimal system of notation, each power of 10 being represented in it by a different figure: $10^5$ ($=100,000$) seems to have been the limit in ordinary use. Each multiple of unity from 1 to 9 had a separate name, and also with the tens from 1 to 90, but the hundreds, as with us, were probably multiples of 100.

The only fractions used were $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, etc., but there was no limit to the divisor: $\frac{1}{5432}$ is given as an example of a high number for a divisor, but it is to be noted that this equals $\frac{1}{7 \times 8 \times 9 \times 11}$, and it does not seem probable that very high prime numbers were used as divisors in calculations.

The system seems to have been very simple—*i.e.*, to break up a fraction into very simple ones, as $\frac{1}{2}+\frac{1}{3}+\frac{1}{8}+\frac{1}{16}$, etc., and this was evidently for the purpose of readily comprehending the amount of the fraction, the smaller fractions probably being thrown away. They were, however, quite accustomed to obtaining least common measures for the divisors, and they were accustomed to treat the denominators as integers.

Thus, from the medical papyrus (Ebers): Prove that

$$1+\tfrac{1}{2}+\tfrac{1}{4}+\tfrac{1}{32}+\tfrac{1}{64}+2+\tfrac{1}{5}=4.$$
$$=3+\frac{160+80+10+5+64}{320}.$$
$$=3+\frac{319}{320}=4 \text{ nearly.}$$

From this it will be seen that they worked within a percentage.

Addition and subtraction, doubling and halving, multiplying and dividing by 10, 2, or 5, were the methods adopted in solving mathematical problems.

Multiplication by numbers other than 2, 5, and 10 was carried out simply by repeated doubling and adding.

Example 1.—Calculate 9 to 6 times $(9 \times 6)$.

(1) 9
(double 9) (2) 18/
(double 18) (4) 36/
(add 2 and 4) (6) 54 (answer).

The dash marked the numbers to be added together.

Division was accomplished by converting the question into a multiplication sum of the same type as the preceding, the divisor being multiplied till the dividend was reached, the answer being the number of times the divisor was so multiplied.

The above examples are taken from a paper by Mr. F. L. Griffiths, 'Notes on Egyptian Weights'; Proceedings of Bib. Arch. Soc., 1892-94, and from 'A Short History of Mathematics.'

## Geometry of the Egyptians.

The deficiency of algebraic symbols and absence of modern decimal notation prevented exactness in calculations, and the ancients had recourse to a give-and-take system, so as to keep

to whole numbers and manageable fractions. The following, however, are modes of calculation which are moderately exact, and which seem to have been in use.

(1) $(126)^3 = 2,000,376.$

(2) $(25\frac{26}{27})^3 = \left(\frac{701}{27}\right)^3 = \frac{344472101}{(27)^3} = 17,500{\cdot}99.$

(3) $7 \times 9^3 = 250,047.$
$(500)^2 = 250,000.$
$2(50)^3 = 250,000.$

(4) $\left.\begin{array}{r}(14)^3 = 2744 \\ \frac{1}{2}(17{\cdot}\frac{64}{100})^3 = 2744{\cdot}5\end{array}\right\}$ This is consequent on (1) as $17\frac{64}{100} = 14 \times 1\frac{26}{100}.$

(5) $(28)^3 = (14)^3 \times 8 = 21,952 = (176 \times 124{\cdot}725).$
Nearly $22,000 = (176{\cdot}4 \times 124{\cdot}725).$
$= (176 \times 125).$

(6) $\left(\frac{25}{14}\right)^2 = \text{nearly} \left(\frac{53}{36}\right)^3.$

(7) $\frac{3}{7} = \frac{1{\cdot}26}{2{\cdot}94} = \sqrt[3]{2} \times \left(\frac{18}{53}\right)$ nearly.

(1) The cube root of 2 is 1·26 approximately. So nearly that if it be multiplied three times together it amounts to 2·000376, and if multiplied twelve times (*i.e.*, on raising the side of a pint cylinder to a quarter cylinder) it amounts to 16·012 instead of 16.

This is probably the Egyptian solution of the ancient problem to find the side of a cube equal to twice the content of a given cube. Given a cube 100 units a side, then its content is 1,000,000, and the content of a cube 126 a side is 2,000,376.

This sum of 126 is made up of $9 \times 2 \times 7$.

If we take a cylinder of 14 height and radius and double its content, its height and radius will now be $14 \times 1{\cdot}26 = 17{\cdot}64$.

If we halve its content, its height and radius will be $\frac{14}{1\cdot26}=11\frac{1}{9}$. Thus we arrive at all the dimensions in the binary system of cylinders, as the remainder can be obtained by halving and doubling.

(2) To show that the cube root of $17{,}500=25\frac{26}{27}=\frac{701}{27}$.

In multiplying $\frac{701}{27}\times\frac{701}{27}\times\frac{701}{27}$ together the Egyptians probably had a short-cut, thus:

$$701\times701=491401.\quad\text{Deduct 1.}$$

$$18200\times\ 27=491400.$$

$$18200\times701=12758200.$$

$$\text{Add 1.}\quad 18199\times701=12757499.$$

$$27\times27\times17500=12757500.$$

$$\therefore\ \frac{701}{27}\times\frac{701}{27}\times\frac{701}{27}=\frac{12757499}{27\times27}=\frac{27\times27\times17500}{27\times27}=17500.$$

$$\therefore\ \left(\frac{701}{27}\right)^3=17500$$

By logs. it $=17{,}500\cdot9$.

It does not seem practicable to treat the double cubit in the same manner; but the Egyptians may have arrived at a near approximation to 70,000 by multiplying $\frac{701}{27}$ by the root of 2 twice over:

$\frac{701}{27}\times1\cdot26\times1\cdot26=41\cdot2188$. This, cubed, gives 70,030.

$\frac{1402}{27}\times\frac{1}{1\cdot26}=41\cdot2110$. This, cubed, gives 69,980.

From this it would have been apparent that their value for the double cubit was very close upon the cube root of 70,000.

| | | |
|---|---|---|
| The true value is | - | $41{\cdot}2128$ |
| And the value taken is | | $41{\cdot}2222 = 41\frac{2}{9}$ |
| | | $\cdot 0094$ |

As this difference is only connected with measures of capacity, and not with linear measure, it is immaterial.

(3) The Egyptians conceived the idea that the fifth root of 17,500 is the square root of $\frac{22}{7}$, which they took to be $\pi$, and they were not very far wrong.*

$$\sqrt{\frac{22}{7}} = 1{\cdot}7728 \qquad \frac{22}{7} = 3{\cdot}14285$$

$$(17{\cdot}500)^{\frac{1}{5}} = 1{\cdot}772585 \qquad (17{\cdot}500)^{\frac{2}{5}} = 3{\cdot}14207$$

$$\sqrt{\pi} = 1{\cdot}772459 \qquad \pi = 3{\cdot}14159$$

They were unable, however, to find any fraction easily used in calculation that was near this value. They therefore had recourse to an artifice by which they reduced the value of $\pi$ in one direction and increased it in another, arriving by this means at 17,500, as follows:

$(17{\cdot}64)^3 \times \frac{25}{14} \times \frac{25}{14}$ represent a cylinder of radius and height 6 palms, or $17{\cdot}64$ inches; but $17{\cdot}64 = 14 \times \sqrt[3]{2}$.

$$\therefore\ (17{\cdot}64)^3 \times \left(\frac{25}{14}\right)^2 = 17{,}500.$$

Or we may take it in another way:

Under p. 33 $(28)^3 = 22{,}000$ nearly, and $\frac{70}{22} = \left(\frac{25}{14}\right)^2$ nearly

$$\therefore\ (17{\cdot}64)^3 \times \frac{70}{22} =$$

$$(14)^3 \times 2 \times \frac{70}{22} =$$

$$\frac{22000}{4} \times \frac{70}{22} = 17{,}500.$$

* It is to be noted that $\left(\frac{22}{7}\right)^{\frac{5}{2}} \times (10)^3$ inches represents a cylinder of $17{\cdot}728$ inches height and radius, with a content of 17,511 cubic inches.

The value of $\pi$ as used in calculation can also be obtained as follows (p. 22):

$$\frac{\text{Cubic inches}}{\text{Cubic palms}} \quad \frac{70,000}{8 \times 2744} = \frac{10,000}{8 \times 392} = \frac{10,000}{3,136} = \frac{25}{14} \times \frac{25}{14}.$$

Thus we get the values of $\pi$ for squaring the circle. We now require the cube root of $\left(\frac{25}{14}\right)^2$. Unfortunately, there is no fraction which quite makes up this amount; the nearest is $\frac{53}{36}$. The difference is:

$$\frac{625 \times (36)^3 \pm (53)^3 \times (14)^2}{(14)^2 \times (53)^3} = \frac{29160000 \pm 29179892}{(14)^2 \times (53)^3} =$$

$$\frac{19892}{(14)^2 \times (53)^3}. \quad \text{Say } \frac{1}{1500}.$$

The palm is $2 \times \frac{53}{36}$ inches, and taking 14 palms we have $41\frac{2}{9}$ inches as the value of the double cubit 41·2222 inches.

The cube root of 70,000 = 41·2128 ,,

·0094

(7) The fraction $\frac{3}{7}$ is the relation between the double cubit and the common cubit: $\left(\frac{3}{7}\right)^3 \times 70,000 =$ common cubit cubed $= 5,510·2$.

This is the value of the common cubit:

$\frac{3}{7}\sqrt[3]{70,000}$ - - $= 17·662$ inches.

The value taken is $17·\frac{2}{3} = 17·666$.

$$\frac{3}{7} = \frac{1·26}{2·94} = \sqrt[3]{2} \cdot \frac{18}{53} \text{ nearly.}$$

The values of these various sums are collected together as follows:

$1{\cdot}26 = {\cdot}7 \times 9 \times {\cdot}2$ (cube root of 2).

$2{\cdot}94 = {\cdot}7 \times 7 \times {\cdot}3 \times 2$ (palm in inches).

$1{\cdot}764 = {\cdot}7 \times 7 \times {\cdot}9 \times {\cdot}9$ ($\frac{1}{10}$ of an ordinary (6 palm) cubit in inches).

$$\frac{1{\cdot}769}{1{\cdot}26} = 1{\cdot}4.$$

$\frac{53}{18} = 2{\cdot}94$ inches (1 palm) (p. 24).

$\frac{53}{36} = \sqrt[3]{\pi E}$ (p. 22).

$\frac{53}{3} =$ common cubit $17\frac{2}{3} = 17{\cdot}666$ inches.

$\frac{53 \times 7}{9} =$ double cubit $41\frac{2}{9}$ inches.

$\frac{701}{27} = 25\frac{26}{27}$ side of cube of $17{\cdot}500$.

Note.—It must not be lost sight of that the ratio of the palm to the primitive inch is as $\frac{53}{18} : 1$ (p. 24): and that a cubic palm is equal in content to a cylinder of 2 inches height and radius.

TABLE (I.) SHOWING THAT WITH A CUBE AND CYLINDER OF EQUAL CONTENT THE SIDE OF THE CUBE IN PALMS IS EQUAL TO HALF OF THE HEIGHT AND RADIUS OF CYLINDER IN INCHES (HEIGHT AND RADIUS OF CYLINDER BEING EQUAL).

| Palms. | | Inches. | Content of the Cubit Cubed in Cubic Inches. | Cylinder of Equal Content to Cube. Height and Radius in Inches. |
|---|---|---|---|---|
| | $\frac{1}{2}$ | $\frac{53}{36}$ | 3·19 | 1 |
| | 1 | $\frac{53}{18}$ | 25·52 | 2 |
| | 2 | $5\frac{8}{9}$ | 204·25 | 4 |
| | 3 | $8\frac{5}{6}$ | 1,636·00 | 6 |
| | 4 | $11\frac{7}{9}$ | 1,636·00 | 8 |
| | $4\frac{1}{2}$ | $13\frac{1}{4}$ | 2,333·33 | 9 |
| Remen* - | 5 | $14\frac{13}{18}$ | — | 10 |
| | $5\frac{1}{2}$ | $16\frac{7}{36}$ | — | 11 |
| Common cubit | 6 | $17\frac{2}{3}$ | — | 12 |
| | $6\frac{1}{2}$ | $19\frac{27}{200}$ | 7,000·00 | 13 |
| Building cubit | 7 | $20\frac{11}{18}$ | 8,750·00 | 14 |
| | $7\frac{1}{2}$ | $22\frac{1}{13}$ | — | 15 |
| | 8 | $23\frac{5}{9}$ | 13,088·00 | 16 |
| 25-inch cubit | $8\frac{1}{2}$ | $25\frac{1}{45}$ | — | 17 |
| | — | $25\frac{26}{27}$ | 17,501·3 | — |
| | 9 | $26\frac{1}{2}$ | — | 18 |
| 28-inch cubit | $9\frac{1}{2}$ | $27\frac{29}{30}$ | — | 19 |
| Double cubit | 14 | $41\frac{2}{9}$ | 70,048·00 | 28 |

* Remen is the name of an Egyptian unit for land measure.

TABLE (II.) SHOWING DERIVATION OF THE DOUBLE CUBIT.

| | | | |
|---|---|---|---|
| $\frac{9068 \cdot 8}{220}$ inches (p. 43) - - | = | Inches.* $41 \cdot 22\dot{1}\dot{8}$ | Used in linear measure. |
| $14 \times \frac{53}{18} = 41\frac{2}{9}$ inches (p. 36) | = | $41 \cdot 2\dot{2}$ | |
| $25\frac{26}{27}\sqrt[3]{2} = 25\frac{26}{27} \times 1 \cdot 26$ (p. 34) | = | 41·2188. | |
| $\sqrt[3]{70{,}000}$ (p. 35) - - | = | 41·21284 | |
| $\sqrt[3]{4(\frac{22}{7})\frac{5}{2}} \times 10^3$ (pp. 35, 54) - | = | 41·2215 | |
| $\frac{2}{3}(7)^2 \times \sqrt[3]{2}$ - - - | = | 41·16 | |
| **THE COMMON CUBIT (pp. 34–36).** | | | |
| $\frac{3}{7}\sqrt[3]{70{,}000} = \frac{6}{7} \times 20 \cdot 6064^2$ | = | 17·66264 inches. | |
| Common cubit in use $\frac{6}{7}(41\frac{2}{9})$ | = | $17 \cdot \dot{6}$ inches. | |
| $17\frac{2}{3}$ | = | | |
| $(41 \cdot \frac{2}{9})^3$ - - - - | = | 70,048 | |
| $(28)^3 \times \left(\frac{25}{14}\right)^2$ - - - | = | 70,000 | |
| $\left(25\frac{26}{27}\right)^3 \times 4$ - - - | = | 70,005·4 | |

* These dimensions are all in primitive inches except the first on the list, which is the measure of the base of the Great Pyramid, and is in British inches. But, as will be seen pp. 47, 53, the British inch and primitive inch are practically identical.

## TABLE (III.) GIVING VARIOUS FRACTIONAL VALUES USED FOR $\pi$ AND $\sqrt{\pi}$ IN EGYPTIAN CALCULATIONS OF MEASURES OF CAPACITY.

| | | | Value of $\pi$. | | Value of $\sqrt{\pi}$. | | Remarks. |
|---|---|---|---|---|---|---|---|
| $\pi b_5$ | $\left(\frac{53}{36}\right)^3 \times \left(\frac{3}{7}\right)^3 70,000 =$ | 17,583 | $\left(\frac{53}{36}\right)^3$ | 3·1902 | The half-palm cubed. | | Used for cylindrical measure. |
| $\pi b_2$ | $\left(\frac{67}{21}\right) \times \left(\frac{3}{7}\right)^3 70,000 =$ | 17,588 | $\frac{67}{21}$ | 3·1904 | | | Equal to $\left(\frac{53}{36}\right)^3$. |
| $\pi b_1$ | $\left(\frac{25}{14}\right)^2 \times \left(\frac{3}{7}\right)^3 70,000 =$ | 17,570 | $\left(\frac{25}{14}\right)^2$ | 3·188775 | $\frac{25}{14}$ | 1·785714 | Used in squaring circle. |
| $\pi b_3$ | $\left(\frac{51}{16}\right)^2 \times \left(\frac{3}{7}\right)^3 70,000 =$ | — | $\frac{51}{16}$ | 3·1875 | | | |
| $\pi b_4$ | $\left(\frac{35}{11}\right) \times \left(\frac{3}{7}\right)^3 70,000 =$ | 17,512 | $\frac{35}{11}$ | $3{\cdot}\dot{1}\dot{8}$ | $\frac{71}{40}$ | 1·775 | |
| $\pi a$ | $\left(\frac{22}{7}\right)^5 \times (10)^3$ - = | 17,510·9 | $\frac{22}{7}$ | 3·1420571 | — | — | Common Egyptian value of $\pi$. |
| Foundation of system, the cylinder of 6 palms | | 17,500·0 | $\frac{1}{10}(17,500)^{\frac{2}{5}}$ | 3·14207 | $\frac{1}{10}\sqrt[5]{17,500}$ | 1·772585 | Nearest Egyptian value of $\pi$. |
| A near approach to $\pi$ - - - | | | $\frac{377}{126}$ | 3·14166 | — | | Not used by the ancients. |
| $(\pi)^{\frac{5}{2}} \times 10^3$ - - - | | 1,749·34 | — | — | $\frac{30}{17}$ | — | |
| | | | $\pi$ | 3·1415926 | $\sqrt{\pi}$ | 1·772459 | |
| The nearest approach to $\pi$ - - | | | $\frac{355}{113}$ | 3·141592 | — | — | |
| A near approach to $\pi$ - - - | | | $\frac{323}{71}$ | 3·140841 | — | — | |
| | | | $\frac{1}{10}$ common cubit of 6 palms. | | $\frac{53}{36}$ | $1{\cdot}7\dot{6}$ | |
| $(25\frac{26}{27})^3$ = cylinder of 6 palms | | 1,7500·0 | — | — | $\frac{3}{70}\sqrt[3]{70,000}$ | 1·76628 | $\frac{3}{7}\sqrt[3]{70,000} = 17{\cdot}6628$. |
| $(1{\cdot}764)^3 \times \left(\left(\frac{25}{14}\right)^2 \times 10\right)^2$ = | | 1,750·33 | — | — | $14\sqrt{2}$ | 1·764 | |
| | | | $\left(\frac{44}{25}\right)^2$ | 3·0976 | $\frac{44}{25}$ | 1·76 | |
| | | | $\frac{31}{10}$ | 3·100 | | | |
| **ORIGINAL VALUES OF $\pi$.** | | | | | | | |
| This may be the origin of common cubit | | | $\frac{19}{6}$ | $3{\cdot}1\dot{6}$ | $\frac{53}{30}$ | $1{\cdot}7\dot{6}$. | |
| $\frac{19}{6} \times \frac{60}{19} = 10$ - - - - - | | | $\frac{60}{19}$ | 3·1578 | $\frac{34}{19}$ | 1·789. | |
| Value of $\pi$ used by Ahmes - - | | | $\left(\frac{4}{3}\right)^4$ | 3·1613 | $\left(\frac{4}{3}\right)^2$ | $1{\cdot}\dot{7}$. | |
| | | | | | $\frac{30}{17}$ | 1·7. | |

# CHAPTER IV

## THE MEASUREMENTS RECORDED IN THE GREAT PYRAMID

The base of the Great Pyramid of Gizeh has been measured many times during the last two centuries with very great care; but it is only in the measurements of Mr. Flinders Petrie that accuracy has been attainable, because he has taken into account the pavement which used to cover up those portions of the Pyramid which lay below a certain level. The Pyramid is built on uneven, rocky ground, the corners are on different levels and the base sides along the surface of the rock are of unequal length, but it is built with such accuracy that on the level the base sides are all equal, and the problem was how to deduce the ancient level of the pavement. This has been successfully accomplished by Mr. Petrie, the base side being now known to have been 9068·8 British inches in length, and the length of the Egyptian cubit can thus be obtained with great accuracy, provided we know the number of cubits the base side measured.

Whether the measurements of the Pyramid by Piazzi Smyth, Flinders Petrie, or any other investigator are examined, it is quite evident to anyone accustomed to building operations that the building cubit was *about* 20·6 inches. This can be deduced from the dimensions of the King's Chamber, 412·25 inches by 206·13 inches (20 × 10 cubits), and from the constant use of multiples of

this number of inches throughout the Pyramid. The length of cubit Petrie deduces from the four sides of the King's Chamber is 20·632±004 inches, but this is not the deduction to be made from his mean measurements, recorded on p. 27, 'Pyramids and Temples of Gizeh.' They run :

$$\begin{array}{r@{}l} & 412{\cdot}40 \\ & 206{\cdot}29 \\ & 412{\cdot}11 \\ & \underline{205{\cdot}97} \\ 60) & \underline{1236{\cdot}77} \\ & \quad 20{\cdot}613 \end{array}$$

giving a cubit of 20·613 inches.

Petrie lays much stress upon the open joints and cracks in the walls of the King's Chamber, and was obliged to deduct the widths of these cracks from the measurements he made to arrive at the original measurements of the chamber. It is, therefore, evidently quite impracticable to get at any very near results from the King's Chamber beyond the fact that the cubit was close on 20·6 inches.

Flinders Petrie's measurements, however, give us a really accurate basis of calculation for the length of this cubit, from the real base of the Pyramid, although he does not use it himself.

Taking, however, anything between 20·60 and 20·65 inches for the length of the cubit, it can be calculated that 440 is the only whole number that will divide into 9,068·8 inches, as the two extremes are 9,064 and 9,077·2 inches, within which neither 439 nor 441 cubits can be obtained. It may then be considered as certain that if the cubit lay between 20·6 and 20·65 inches, the base sides measure each 440 Egyptian cubits.

THE BASE AND HEIGHT OF THE GREAT PYRAMID.*

This base is 22 times the length of the King's Chamber, and therefore there is a prospect we may get from it the cubit length to another place of decimals accurately, and moreover, the settlements in the Pyramid would not affect

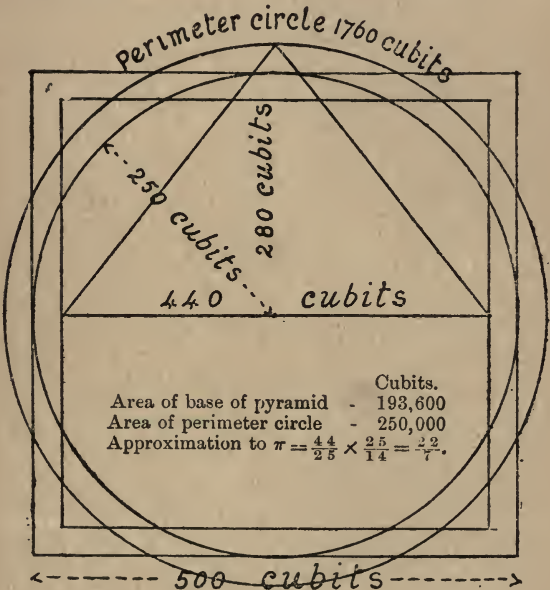


FIG. 5.

materially the masonry near the rock on which the base is built. The length of base, 9,068·8 inches, divided by 440, gives a cubit of 20·61$\dot{0}\dot{9}$ inches. Petrie gives the mean value of base as true within half an inch, thus allowing of an extreme range for the cubit of from 20·6097 to 20·612 inches, and giving the cubit length to within ·0023 inch. It does

* See p. 22.

not seem practicable to get a nearer approximation to the ancient cubit than this. The cubes of the doubles of these three sums are:

$$(41{\cdot}2194)^3 = 70{,}033{\cdot}4 \text{ inches.}$$
$$(41{\cdot}2218)^3 = 70{,}045{\cdot}7 \quad ,,$$
$$(41{\cdot}224)^3 = 70{,}056{\cdot}8 \quad ,,$$

Before, however, accepting the $20{\cdot}61\dot{0}\dot{9} \pm {\cdot}00115$ inches as the exact length of the cubit, let us make quite sure that Petrie measured between the right points.

The rock-sockets, in which were the corner-stones of the Pyramid, are known: and previous to the measurements of Petrie the lengths of the Pyramid sides were taken from socket to socket. But as these sockets are on different levels, no really accurate results could be obtained.

Petrie has shown clearly that these sockets (although they held the corner-stones), do not indicate the terminals of the base sides, as there was a level limestone pavement some inches above, and that the actual base of the Pyramid, as exposed to view when it was completed, was on one level at the surface of this pavement. The accuracy of the length of base given by Petrie must, therefore, depend upon the level at which he puts this pavement.

His measurements from socket to socket (9,125·9 inches taken as a mean) accord closely with the best of the later measurements, lying between 9,120 of the Ordnance Surveyors and 9,142 of Piazzi Smyth. We may therefore have confidence in his deduction, provided he assumed a right level for the pavement and a correct angle for the slope of the Pyramid.

About this latter there can be no doubt. As will be shown, the slope is 28 perpendicular to 22 horizontal, so that a mistake in the level of the pavement of an inch would cause an increase or decrease in the length of the base side of about 1·4 inches.

Petrie had the work of the former explorers to guide him concerning the level of this pavement, and there is also still existing a magnificent basalt pavement covering more than a third of an acre close at hand, which must have been very nearly on a level with the limestone pavement around the Pyramid. His calculation arrives at the result that this basalt pavement was 2 inches above the level of the pavement around the Pyramid. This basalt pavement was taken by him as zero-point, and the socket levels with reference to it are: N.E., $-28{\cdot}5$; S.E., $-39{\cdot}9$; S.W., $-23{\cdot}0$; and N.W., $-32{\cdot}8$ inches. From these data, and taking into consideration the distances of the casings from the edges of the sockets, varying from 4·8 to 12·6 inches (Plate VI., 'Pyramids and Temples of Gizeh'), I have recalculated the lengths of the base sides, and find them to be 9,069·1, 9,067·4, 9,069·9, and 9,068·7 inches, giving a mean of 9,068·72 inches, varying less than $\frac{1}{10}$ inch from that of Petrie. I think, therefore, his mean length of base side may be accepted with confidence as 9,068·8 inches, giving a cubit of $20{\cdot}61\dot{0}\dot{9}$ inches $\pm$ ·00115; but yet it must not be lost sight of that an error of 1 inch in length of base side either way would increase or reduce the length of the cubit from 20·613 to 20·6086, equal to about 2 in 10,000.

As for the height of the Pyramid, all recent authorities are agreed that it bore to the base about the proportion which the radius of a circle has to the circumference, or $\frac{22}{7}$.

The height has been estimated by the angle still existing on many of the stone casings, and there is little or no difference of opinion on the subject.

P. Smyth's estimate is from 51° 50′ to 51° 52′ 18″, and Petrie's 51° 52′ ± 2′.

P. Smyth proposes that the Egyptians used for $\pi$ the proportions 116·3 and 366, giving 3·14703, while $\frac{22}{7}$ gives 3·1428.

Petrie calculates a height of 5,776·6 ± 7 inches, which at 280 cubits in height would give a cubit of 20·628 inches, but his large limit of error would allow the cubit to range between 20·607 and 20·645, and as these limits will not allow of either 279 or 281 cubits, I think it is evident that the height should be 5,770·54 inches, or 280 cubits of $20 \cdot 61\dot{0}\dot{9}$ inches. Petrie proposes that the builders used the proportion $\frac{22}{7}$ for $\pi$, which must necessarily follow from the proportions of the height, 280 to 440 base side.*

We now have before us the problem already discussed at p. 21—two circles derived from a square of 44 a side whose area and radii are whole numbers.

In the Pyramid base we have 440 cubits.

| Circumference. | Radius. | Side. | Area. |
|---|---|---|---|
| $\frac{11,000}{7} = 2 \cdot \frac{22}{7} \times 250$ | | 440 | $\pi c r^2 = \frac{44}{25} \cdot \frac{44}{25} \cdot 250 \cdot 250 = 193,600.$ |
| $1,760 = 2 \cdot \frac{22}{7} \times 280$ | | 500 | $\pi b r^2 = \frac{25}{14} \cdot \frac{25}{14} \cdot 280 \cdot 280 = 250,000.$ |

If the two circles derived from a square on 440 be made into cylinders by multiplying each by the radius of the perimeter circles we have for the perimeter circles:

$$\frac{22}{7} \quad - \quad - \quad 250,000 \times 280 = 70,000,000,$$

$$\pi \quad - \quad - \quad 250,000 \times 282 \cdot 09 = 70,522,500,$$

or a difference of $\frac{1}{140}$. Again, in the area circle multiplied by the same radii we have:

$$\frac{22}{7} \quad - \quad - \quad 193,600 \times 280 = 54,208,000,$$

$$\pi \quad - \quad - \quad 196,347 \times 282 = 55,388,535 \cdot 23,$$

or a difference of $\frac{1}{50}$, or 2 per cent.

* Sir Henry James, in his notes on the Great Pyramid, published in a pamphlet in 1869, points out that the corner lines rise 9 units in height for every 10 units of horizontal distance along the diagonals. This would be so for all practical purposes with the proportion $\frac{22}{7}$ for $\pi$.

The whole content of the Great Pyramid is $\frac{54,208,000}{3} =$ $18,069,333\cdot\dot{3}$ cubic cubits. If these dimensions be reduced by $10 \times 20\cdot6014$ per side, we have a small pyramid of base 44 pyramid units; in fact, a miniature of the Great Pyramid.

| Circumference. | Radius. | Side. | Area. |
|---|---|---|---|
| $\frac{110}{7}$ | 25 | 44 | 1,936 |
| 176 | 28 | 50 | 2,500 |

All expressed in pyramid units.

Now, the length of the cubit in British inches is - $20\cdot61\dot{0}\dot{9}$

And in pyramid units $\frac{1}{2}\sqrt[3]{70,000}$ - - - - $20\cdot6064$

Giving a difference between the two cubits of - $\cdot0045$

But as fractions only could have been used the nearest approach is $20\frac{11}{18}$ primitive units to a cubit, which equals $20\cdot61111$ primitive units, so that the difference is only $\cdot0002$ or 1 in 100,000; thus practically they are identical.

### The Measurements of the Pyramid Coffer.

The proportions I now put forward do not depend upon Pyramid measurements. They can be deduced as the solution of a problem: and, given the length of the base of the Pyramid in inches, the correct measurements of the coffer can be given by calculation. In other words, we have now the information that the builder of the Pyramid had when he designed the work, and we can give the dimensions which the workmen carried out. So that a coffer can be constructed in this country of the exact measurements given in the design for the original coffer, and the coffer measurements of recent surveys can be tested.

If the base side of the Pyramid has not been given quite correctly, the only effect on the view now put forward would be to alter the length of the cubit and to alter the relation of

the pyramid unit to the British inch in a very minute degree, but in other respects there would be no alteration.

## The Problem Stated.

Assuming that the dimensions and proportions of the Great Pyramid indicate the knowledge* possessed by the Egyptian wise men as to circular measure it is desired to embody this knowledge in one vessel of stone, which shall also be a record of their knowledge of the harmonical progression and of the volume of a sphere.

First, as regards the musical or harmonical progression. Pythagoras discovered, or rediscovered, that with a vibrating string the lengths which give a note, its fifth and its octave, are in the ratio $1 : \frac{2}{3} : \frac{1}{2}$, or $6 : 4 : 3$. This, it appears, was known ages before to the Egyptians and recorded in the stone coffer.

Ahmes, the Egyptian, in the papyrus (already referred to) of the Rhind Collection, tells us of a barn or box whose linear dimensions are in terms of $a$, $b$, $c$, as follows: $a \times b \times (c \times \frac{1}{2}c)$. This is one clue. Put this in form of a harmonic progression, and it becomes:

$$a \times b \times (c + \tfrac{1}{2}c).$$
$$3 \times 4 \times (6 + 3).$$

What is desired is to make a coffer of this shape, dimensions in palms, to hold the content of four pyramids each

$$\frac{(7\frac{1}{2})^2 \times 9\frac{1}{2} \text{ palms}}{3} = \frac{2850}{4} \text{ cubic palms.}$$

Multiply each factor of the harmonic progression by 3, and it becomes:

| Breadth. | | Height. | | Length. | | |
|---|---|---|---|---|---|---|
| 9 | × | 12 | × | 27 | = | 2,916. |

Thus being 66 in excess of the required amount.

* See pp. 21, 44, and 47.

The bulk or solid portion of this coffer is to represent $(14)^3 = 2{,}744$ cubic palms.

Giving 2 palms to the thickness of sides, we have for outside length 31, outside breadth 13, and, if the bottom thickness be $2\frac{1}{4}$, the outside height would be $14\frac{1}{4}$.

The volume over all would therefore be:

$$\begin{array}{r} 31 \times 13 \times 14\frac{1}{4} = 5{,}742\frac{3}{4} \\ 2{,}850 + 2{,}744 = \underline{5{,}594} \\ 148\frac{3}{4} \end{array}$$

$$\begin{array}{l} 5{,}742\frac{3}{4} \\ \underline{2{,}916} \\ 2{,}826\frac{3}{4} \text{ less } 82\frac{3}{4} = 2{,}744. \end{array}$$

So that the proportion runs:

| | | |
|---|---|---|
| Bulk | ... ... | $2{,}744 + 82\frac{3}{4}$ |
| Interior | ... ... | $2{,}850 + 66$ |

Thus, then, there is rough approximation to the proportion required, as near as can be taken with palms.

| | | | |
|---|---|---|---|
| Volume over all | ... | $31 \times 13 \times 14\frac{1}{4}$ | $= 5{,}742\frac{3}{4}$ |
| Content | ... ... | $27 \times 9 \times 12$ | $= 2{,}916$ |
| Solid bulk ... | ... | ... ... | $= 2{,}826\frac{3}{4}$ |
| Bottom | ... ... | $31 \times 13 \times 2\frac{1}{4}$ | $= 906\frac{3}{4}$ |
| Sides | ... ... | $\left\{ \begin{array}{l} 2 \times 31 \times 12 \times 2 = 1{,}488 \\ 2 \times 9 \times 12 \times 2 = 432 \end{array} \right\} 1{,}920$ | |

This is suggested as the first approach to the shape of the pyramid coffer, and if it is put into inches it will be found to closely approximate, but is about 2 per cent. too large.

### The Pyramid Coffer Problem.

To construct a rectangular coffer whose content is

$$\begin{array}{rl} & 72{,}333\dot{3} \text{ cubic inches.} \\ \text{whose bulk is} & \underline{70{,}000\dot{0}} \quad ,, \quad ,, \\ \text{with volume over all} & 142{,}333\dot{3} \quad ,, \quad ,, \end{array}$$

The measurements of contents to be in the terms of musical or harmonic progression, 6, 4, 3, changed as indicated by Ahmes in the Rhind papyrus according to the formula $a \times b \times \left(c + \frac{c}{2}\right)$ to 9, 4, 3, and further changed to 9, 4, $\pi c$ $\left(= \frac{44}{25} \times \frac{44}{25} = 3{\cdot}0976 = \text{ say } 3{\cdot}1\right)$.

Thus:

The sides of the coffer are to have about double the bulk of the bottom:

*i.e.*, the four sides are to have a bulk of $46{,}666{\cdot}\dot{6}$ cubic ins.

The bottom is to have a bulk of $23{,}333{\cdot}\dot{3}$ „

Total bulk ... $70{,}000{\cdot}0$ „

The length, breadth, and height of the interior of the coffer are to be in the following proportion: Length, 9; breadth, $\frac{31}{10}$; height, 4; thickness of sides, $\frac{2}{3}$; thickness of bottom, $\frac{4}{5}$.

The relation of the bulk (70,000) to content ($72{,}333{\cdot}\dot{3}$) is as 3 to 3·1, so that $70{,}000 = \frac{3{\cdot}1}{3} \times 72{,}333{\cdot}\dot{3}$,

3·1 being the assumed ratio of circumference of a circle to diameter 1, in use when the coffer was constructed.

The dimensions of the coffer are required in primitive inches.

---

$$\text{Now } 9 \times 4 \times \frac{17{,}500}{9} = 70{,}000.$$

$$\therefore\ 9 \times \tfrac{31}{10} \times 4 \times \frac{17{,}500}{27} = 72{,}333{\cdot}3.$$

$$\text{Also } \frac{17{,}500}{27} = \tfrac{1}{3}\sqrt[3]{17{,}500} \times \tfrac{1}{3}\sqrt[3]{17{,}500} \times \tfrac{1}{3}\sqrt[3]{17{,}500}.$$

∴ The dimensions of interior of coffer are :

Length. Breadth. Height.

$$(9 \times \tfrac{1}{3})\sqrt[3]{17{,}500} \times (\tfrac{31}{10} \times \tfrac{1}{3})\sqrt[3]{17{,}500} \times (4 \times \tfrac{1}{3})\sqrt[3]{17{,}000} = 72{,}333 \cdot \dot{3}.$$

$$\text{Now } \tfrac{1}{3}\sqrt[3]{17{,}500} = 8 \cdot 654136 = 8\tfrac{53}{81} \text{ nearly.}$$

∴ Dimensions of interior of coffer in inches are :

| Length. | Breadth. | Height. | |
|---|---|---|---|
| $77 \cdot 887224 \times$ | $26 \cdot 827825 \times$ | $34 \cdot 616544$ | $= 72{,}333 \cdot \dot{3}$ |
| $77\frac{8}{9}$ | $\times\, 26\frac{33}{40}$ | $\times\, 34\frac{3}{5}$ | $= 72{,}300$ |
| | Discrepancy ... | | $33 \cdot 3$ |

*Bulk of Sides.*

Two longer sides :

Length. Breadth. Height.

$$(9 \times \tfrac{1}{3})\sqrt[3]{17{,}500} \times (\tfrac{4}{3} \times \tfrac{1}{3})\sqrt[3]{17{,}500} \times (4 \times \tfrac{1}{3})\sqrt[3]{17{,}500} = \quad 31{,}111 \cdot 1$$

Two shorter sides :

Length. Breadth. Height.

$$\left\{(\tfrac{31}{10} + \tfrac{4}{3}) \times \tfrac{1}{3}\right\}\sqrt[3]{17{,}500} \times (\tfrac{4}{3} \times \tfrac{1}{3})\sqrt[3]{17{,}500} \times (4 \times \tfrac{1}{3})\sqrt[3]{17{,}500} = 15{,}325$$

| | |
|---|---|
| Total bulk of sides ... | 46,436 |
| Bulk required ... ... | 46,666·6 |
| Discrepancy ... | 230 |

*Bulk of Bottom.*

Length. Breadth. Height.

$$(\tfrac{31}{2} \times \tfrac{1}{3})\sqrt[3]{17{,}500} \times \left\{(\tfrac{31}{10} + \tfrac{4}{3}) \times \tfrac{1}{3}\right\}\sqrt[3]{17{,}500} \times (\tfrac{4}{5} \times \tfrac{1}{3})\sqrt[3]{17{,}500} = 23{,}753$$

| | |
|---|---|
| Bulk required ... ... | 23,333 |
| Discrepancy ... | 400 |

| | |
|---|---|
| Bulk of sides ... ... ... | 46,436 |
| Bulk of bottom ... ... ... | 23,753 |
| Total bulk ... ... ... | 70,189 |
| Bulk required ... ... ... | 70,000 |
| Discrepancy ... | 189 |

*Volume of Coffer over all.*

| Length. | Breadth. | Height. |
|---|---|---|

$$\left(\tfrac{31}{3} \times \tfrac{1}{3}\right)\sqrt[3]{17{,}500} \times \left\{\left(\tfrac{31}{10} + \tfrac{4}{3}\right) \times \tfrac{1}{3}\right\}\sqrt[3]{17{,}500} \times \left(\tfrac{24}{5} \times \tfrac{1}{3}\right)\sqrt[3]{17{,}500} = 142{,}520$$

| | | |
|---|---|---|
| Volume required | ... ... | 142,333 |
| Discrepancy | ... | 177 |

Comparison of dimensions thus obtained with measurements of Petrie and Smyth in inches :

| | Volume over all. | | | Within. | | | Thickness. | |
|---|---|---|---|---|---|---|---|---|
| | Length. | Breadth. | Height. | Length. | Breadth. | Height. | Side. | Bottom. |
| Warren | 89·425<br>$89\frac{17}{40}$ | 38·365<br>$38\frac{11}{30}$ | 41·54<br>$41\frac{27}{50}$ | 77·888<br>$77\frac{8}{9}$ | 26·827<br>$26\frac{33}{40}$ | 34·616<br>$34\frac{3}{5}$ | 5·769<br>$5\frac{31}{40}$ | 6·92<br>$6\frac{37}{40}$ |
| Petrie ... | 89·62 | 38·50 | 41·31 | 78·08 | 26·85 | 34·43 | 5·89<br>5·67 | 6·89 |
| Smyth ... | 89·71 | 38·65 | 41·17 | 77·93 | 26·73 | 34·34 | 5·99 | 6·92 |

Comparison of bulk and content in cubic inches :

| | Content. | Bulk. | Volume over all. | Two Long Sides. | Two Short Sides. | Sides. | Bottom. |
|---|---|---|---|---|---|---|---|
| Warren : Calculated | 72,333·3 | 70,000 | $142{,}333{\cdot}\dot{3}$ | — | — | 46,666·6 | 23,333·3 |
| Practical | 72,300·0 | 70,199 | 142,500 | 31,111·1 | 15,325 | 46,436·0 | 23,753 |
| Petrie ... | 72,030 | 70,500 | 142,530 | — | — | 46,667 | 23,830 |
| Smyth ... | 71,317 | 70,999 | 142,316 | — | — | 47,508 | 23,758 |

If we now test the measurements of Petrie and Smyth by the calculations I have recovered, it will be seen that Petrie's measurements accord closely with those calculated, but that there are insufficient data for a mean of measurements for height,

there being only one point available in the top of the broken coffer (see 'Our Inheritance in the Great Pyramid'). The difference is 300 cubic inches, or about $\frac{1}{240}$; the solid bulk being about 300 too much and the content 300 too little, the volume over all agreeing almost exactly. Of course, the discrepancy may be due to the workmen having failed to work exactly according to the measurements given them, and thus having made the coffer not quite deep enough. There is a good deal of rough work, however, in the hollowing out of the interior of the coffer, and the best test is the comparison of the external dimensions.

Note.—In these tables the dimensions given by Petrie and Smyth are in British inches, while my calculations are in primitive inches; the difference, however, is too small for correction. For example (as shown p. 47), the difference is about 1 in 100,000—*i.e.*, 100,001 British inches equal 100,000 primitive inches. No correction, therefore, can be made.

TABLE (IV.) SHOWING THE VARIOUS MEASURES REPRESENTED BY THE PYRAMID COFFER.

| | Dimensions in Linear and Cubic Inches. | Cubic Inches by Egyptian Value of $\pi$. Quarter. | Cubic Inches by Egyptian Value of $\pi$. Chest. |
|---|---|---|---|
| Bulk. | Bulk of stone coffer, bottom $23{,}333{\cdot}\dot{3}$, sides $46{,}666{\cdot}\dot{6}$ C.I. ... | — | 70,000 |
| | Cylinder $(28)^3 \times \pi b \left(=\frac{25}{14}\cdot\frac{25}{14}\right)$ (true content by $\pi = 68{,}962$ C.I.) | — | 70,000 |
| | Box, $50 \times 50 \times 28$ ... ... ... | — | 70,000 |
| | 4 cylinders $= 4\left(\frac{22}{7}\right)^{\frac{5}{2}} \times (10)^3$ (pp. 35 and 39) ... ... .. ... | 17,511·04 | 70,044·16 |
| | Cube of 41·2128, the double cubit of 14 palms ... ... ... | — | 70,044·16 |
| | 4 cubes on $25\frac{26}{27}$ ($\sqrt[3]{17{,}500} = 25{\cdot}95$) | 17,576·0 | — |
| Content. | 4 Pyramids $\frac{(22)^2 \times 28}{3}$ ... ... | 18,069·33 | 72,277·33 |
| | 4 cones $\frac{(25)^2 \times 28 \times \pi c}{3} = \left(\frac{44}{25} \times \frac{44}{25}\right)$ | | |
| | Sphere $\frac{4}{3}\pi c \frac{70{,}000}{4}$, *i.e.*, a sphere of radius $\sqrt[3]{17{,}500}$ = nearly $25\frac{26}{27}$ ... | — | 72,277·33 |
| | Coffer of harmonic progression : $77\frac{8}{9} \times 26\frac{33}{40} \times 34\frac{3}{5}$ $\left(3 \quad \times \quad \frac{4}{3} \quad \times \pi c \frac{1}{3}\right) 17{,}500$ | — | 72,277·33<br>$72{,}333{\cdot}\dot{3}$ |

# CHAPTER V

## EARLY MEASURES OF EGYPT

The base of the Great Pyramid by F. Petrie's measurements is 9068·8 inches.

440 cubits of $41\frac{2}{9}$ primitive units amount to the same number nearly ... ... $9068{\cdot}\dot{8}$ inches.

It may be purely accidental that our British inch is so exactly the length of its progenitor; but whether subsequent measurement of the Pyramid base should show that there is any small difference or not, it can only affect the primitive unit in a very minute degree, and the fact still remains that the British inch closely corresponds to that of Egypt.

It happens also that the length of the base of the Pyramid closely coincides with one-eighth of a minute of arc of the meridian in latitude 30° N. This may, again, be purely accidental.

| | |
|---|---|
| Minute of latitude 30° N. = 60 × 101·67 feet | = 73202 inches. |
| Eight times base of Pyramid ... ... | = 72548 „ |
| Difference about $\frac{1}{100}$ ... | 653 „ |

There are thus nearly 1,760 double cubits ($41\frac{2}{9}$ inches) in a minute of arc of the meridian, just as there are 1,760 British double cubits (yards) in a mile, (the yard being nearly

$41\frac{2}{9}\sqrt[3]{\frac{2}{3}} = \sqrt[3]{46{,}698} = 36{,}010$ inches). There are also 1,760 yards in a minute of arc of longitude close to latitude 30° N.

Our British 10 acres is the square on 10 chains or 220 yards, and is apparently our most ancient standard measure of surface. We also find in Egypt a square of 220 double cubits as the superficial measure there (in the square base of the Great Pyramid), and we may take it to be the original standard of superficial measure.

At the time the Rhind papyrus was written (1600 B.C.) this ancient standard had given way to one in which the multiple of 11 had disappeared, and there seems no sign of it in Greek and Roman measures. It is most interesting to find it still surviving in Great Britain in our linear and superficial measures.*

The original Egyptian standard measure of capacity among the ancients was a cylinder of one common cubit height and radius, with a content of 684 cubic palms, the cubit being 6 palms in length

and the value of $\pi$ being taken at $\frac{19}{6}$.

$$\frac{19}{6} \times (6)^3 = 684 \text{ (p. 9)}.$$

When the Egyptians (before the building of the Pyramid) adopted the second or subsidiary unit—the primitive inch —they arrived at their results as follows: They were aware that the cube root of 2 is $1\frac{13}{50}$ nearly exactly, and that a cubit of $17\frac{16}{25}$ inches is $14 \times \sqrt[3]{2}$ inches (pp. 26, 36).

Their calculations for the content of the cylinder of 6 palms height and radius would therefore be:

$$\frac{25}{14} \times \frac{25}{14} \times (14\sqrt[3]{2})^3 = 17{,}500 \text{ cubic inches.}$$

At first they only subdivided the height and radius of

* For the use of the unit 220 in our early land measures in Anglo-Saxon times, see 'The Tribal Hidage' ('Trans. Rl. Hist. Soc.,' 1900).

the cylinder, and their measures descended in capacity by division by eight as our *principal* British measures (quarters, bushels, gallons, and pints) do at the present day. For the sake of simplicity and identification they are in the following Table V. called by our English names, and are compared with our Winchester and imperial measure:

| | English Designation. | Cubic Inches. | Cylinder* Height and Radius in Palms. | Cubic Inches.† | |
|---|---|---|---|---|---|
| | | | | Imperial Measure. | Winchester Measure. |
| | 4 quarters | 70,000 | $9\frac{1}{2}$ | 70,982·09 | 68,806·41 |
| | Quarter | 17,500 | 6 | 17,745·52 | 17,201·6 |
| | Bushel | 2,187·5 | 3 | 2,218·19 | 2,150·2 |
| Hekt | Gallon | 273·4 | $1\frac{1}{2}$ | 277·27 | 268·7 / 272·2 |
| | Pint | 34·17 | $\frac{3}{4}$ | 34·61 | 34·03 |
| Hon | Pound | 27·34 | Nearly 3 inches cube | 27·72 | 27·22 |

Of these, the only names that have come down to us are the hekt for the gallon and the hon for the pound, or $\frac{1}{10}$ hekt.

At the rate of 252·458 imperial† grains to a cubic inch, of distilled water this would give the weight of the Egyptian gallon 69,021·13 grains imperial and of the hon 6902·11 imperial grains.

The following tests are given as to the accuracy of this deduction: In 'Notes on Egyptian Weights and Measures' ('Proc. Bib. Arch.,' 1892-94), Mr. F. L. Griffiths, quoting

* See pp. 10, 12, and 26.

† These measures are taken from pp. 11, 12, and 88, 'Our Weights and Measures' (Chaney).

from the Rhind and Ebers papyri and other Egyptian records, gives the proportions:

| | | | |
|---|---|---|---|
| | Hinnu or hon | ... ... | = 50 kat or kati. |
| | | | = 5 utens of water. |
| (Gallon) | Hekt | ... | = 10 hon. |
| (Dell or tovit) | Apit | ... | = 4 hekt. |

He estimates that the uten weighed from 1,400 to 1,500 grains imperial, and that the kat from the beginning of the new kingdom weighed from 140 to 150 grains imperial.

The lower value of the kat (140 grains imperial) is the same as that given in Madden's 'Jewish Coinage' (p. 277), derived from a stone weight of 5 kat, weighing 688 grains imperial (originally 700 grains imperial ?).

Madden also states that the copper coins of the Ptolomies follow the same standard as that of the ancient Egyptians, and gives five examples of weights which are multiples of 70 grains imperial.

Moreover, the only gold Egyptian coin in Mr. Griffiths' list older than the Seventh Dynasty is one of 10 units, weighing 2,060 grains imperial—*i.e.*, 15 kats of 137·2 grains imperial.

The Vicomte François de Salignac Fénelon, in his recent work on 'Bimetallisme chez les Hébreux,' also quotes a weight found at Jerusalem weighing 42,000 grains imperial.

Professor Fritz Hommel, in his article 'Babylonia' (Hastings' 'Dictionary of the Bible'), also alludes to a silver shekel of 9·1 grammes, which 'answers exactly to the ancient Egyptian kat of 9·1 grammes' (140·51 grains imperial). He also states that 10 kat made up the Egyptian uten of 1405·1 grains imperial. The greater number of the Egyptian coins of the 140 imperial grain system seems to run from 138 to 139 grains imperial.

There seems, therefore, concurrent testimony that the

ancient weight of the Egyptian kat was about 140 grains imperial, or a little less. This would give to the Egyptian pound or hon a weight of from 6,900 to 7,000 grains imperial, the value of the hon as given above (27·34 cubic inches) being 6902·1 grains imperial. We may therefore look with confidence upon the Egyptian hekt, or gallon, as measuring about 273·4 cubic inches, and the hon as measuring about 27·34 cubic inches, in the early times of the Egyptian Empire, before the Seventh Dynasty.

### Later Egyptian Measure.

In comparatively later times the binary system was introduced—*i.e.*, measures each half or double of the other; thus were added measures corresponding to our quart and pottle, respectively 2 and 4 pints; the peck and dell, or tovit, respectively 2 and 4 gallons; the strike and combe, respectively 2 and 4 bushels. The only Egyptian designation which seems to have come down to us are the apit, corresponding to our dell, or tovit, and the ram, corresponding to our strike.

Mr. Griffiths, in his 'Notes,' deals with the Rhind papyrus and weights of a much later period—about 1600 B.C., when the binary system had been introduced many years—and he adopts 1,500 in preference to 1,400 grains imperial to the uten, on account of a difficulty which arises in the Rhind papyrus as to the number of hon in a cubit cubed (which cubit he takes as between 20·6 and 20·65 inches). He considers the papyrus to show that there are 300 hon to a cubit cubed and 200 hon to a khar or $\frac{2}{3}$ (cubit cubed); whereas, according to the weights of the earlier period, there appear to have been 320 hon to a cubit cubed and 213·3 hon to a khar.

No doubt there is a difficulty on the subject at this later time (1600 B.C.), but it seems that the taking of 300 instead of 320 introduces a still greater difficulty if the cubit of 20·64 inches is to be adhered to, as it leads to the adoption

of weights and measures whose sides (if shaped as cylinders) cannot be represented by whole numbers or manageable fractions of palms or inches. In this difficulty Mr. Griffiths adopts 29·1 cubic inches to a hon in preference to 27·34 cubic inches, as given above.

It seems probable that the Egyptians in these later times would prefer to use an incorrect multiple (viz., 300 and 200 for 320 and 213·3) in their calculations than to have sides to their cylinders and cubes which could not be measured by either palms or inches.

The following values are, then, to be attached to the various measures above quoted (the cubit $20\frac{11}{18}$ inches):

| | Side of Cube. |
|---|---|
| Cubic cubit = 8,750 cubic inches = 320 hon | 1 cubit. |
| The khar or $\frac{2}{3}$ (cubit)$^3$ = 5,833·$\dot{3}$ cubic inches = $\frac{1}{3}$ (640) hon ... ... ... ... | 18 inches. |
| The apit or ($\frac{1}{2}$ cubit)$^3$ = 1,093·75 cubic inches = 40 hon ... ... ... ... ... | $\frac{1}{2}$ cubit. |
| The hekt (or gallon) = 273·4 cubic inches = 10 hon ... ... ... ... ... | $6\frac{1}{2}$ inches. |
| The hon or hennu = 27·34 cubic inches ... | 3 ,, |
| The uten = 5·46 cubic inches = $\frac{1}{5}$ hon ... | 1·75 ,, |

It is to be noted that the khar of $\frac{2}{3}$ (cubit)$^3$ is equal to $\frac{1}{4}$ of the kor or homer (Hebrew), which measures $\frac{1}{2}$ (36 inches)$^3$. See Hebrew Measures.

Mr. Griffiths gives 5,827·88 cubic inches for the third of the cubit of 20·6 + inches cube, as against 5,833·3 cubic inches here given, showing that there is a very close agreement in the value of the cubit he had adopted.

*Note.*—The term $\frac{2}{3}$ cubit cubed may be interpreted in two ways, either as $\frac{2}{3}$ (cubit)$^3$ or as ($\frac{2}{3}$ cubit)$^3$, the former being to the latter as 1 : $\frac{4}{9}$. The former has been accepted as the meaning of the expression in the Rhind papyrus.

## Ancient Grains.

We may now advance a step in deducing the weight of the ancient grain by inquiring into the number of ancient grains in a kat of 138 to 140 grains imperial, assuming that the ancient grain was somewhat heavier and certainly not lighter than our grain imperial.

Mr. R. S. Poole (Weights, Smith's 'Dictionary of the Bible') lays down as one of seven postulates: 'All the older systems are divisible either by 60,000 or by 3,600.' We may therefore expect to find that there were 120 grains ancient in a kat, and we will endeavour to ascertain whether there are any indications of the numbers originally adopted.

In the original measures, based on a cylinder of 6 palms a side, it has been pointed out in Chapter I. that the smallest measure was likely to have been a cube of a palm a side. Now, a palm (2·94 inches) cube contains 25·5 cubic inches, and holds water weighing 6,476 grains imperial, and we know by practical experiments, referred to hereafter, that water bears to pressed barley in weight the ratio of 7 : 5; therefore a palm cube of barley by weight would hold 4625·5 grains imperial. Assuming, then, that ancient grains were heavier than grains imperial, and that the number was likely to be a round number, we may hazard the supposition that the original number was the first round number below 4625·5, viz., 4,000; *i.e.*, that 4,000 ancient grains of barley could be crammed into a cube of a palm a side. This would give 5,600 ancient grains' weight if the same cube were filled with water.

Now, it so happens that this gives to a hon (27·3 cubic inches) of water close upon a weight of 6,000 (6,006) grains ancient, and to a cubic inch of water 220 grains ancient. A hon is 50 kat, and therefore a kat would weigh 120·3 grains ancient, which is close on the number already surmised as

probable. This gives 15,400,000 grains ancient for 70,000 cubic inches of water, the weight of 4 quarters, and 11,000,000 grains ancient for the number of grains ancient contained in 4 quarters. If the kat is taken at exactly 120 ancient grains, we have 15,360,000 ancient grains instead of 15,400,000 to a double cubit cubed. This estimation of the weight of ancient grains is, of course, purely conjectural, but on turning to Babylonian and Hebrew weights and measures it will be found that the same results are arrived at in another manner.

### The Length of a Second of Arc of Latitude and Longitude on the Earth's Surface.

If we take the lengths of seconds of latitude and longitude in feet on the surface of the earth (the compression being taken at $\frac{1}{300}$), at latitude 30° N., we shall find them respectively to measure 101·67 and 87·90 feet.

If we suppose the second to be divided into 60 parts or units, those units will measure respectively 20·334 and 17·580 inches, and their ratio to each other will be as 7 : 6·05.

At latitude 30° 4′ N. at the present time they would be exactly in the proportion of 7 : 6, viz., 20·337 and 17·4318.

The question arises: Could the ancient Egyptians have measured on the earth's surface seconds of latitude and longitude, and if so, with what exactitude could it have been carried out?

With regard to longitude there seems no great difficulty. All that is necessary is to erect planes perpendicular to the surface of the earth, running north and south on the same latitude, at a convenient distance from each other—say at the difference of transit of sun or star of 4 minutes of arc or 16 seconds of time. This distance could be arrived at by trial at latitude 30° N.; it would be about 4 miles, and could be deduced with the assistance of a good carpenter of the present

day with an error of not more than 1 per cent. for the constructional work, but errors in judging the difference of time might seriously effect the computation.

For latitude the difficulty would be greater, as the operation would involve taking the angle of elevation of the Pole Star at its upper and lower culminations, or of the sun at noon at the summer and winter solstices, at two places on a meridian, and the calculation of an angle was not an easy matter in early days. The builders of the Pyramid would, however, have found nothing insurmountable* in this matter, and could probably have arrived at the truth with considerable accuracy.

The Pyramid of Gizeh is situated about 1′ 10″ south of latitude 30° N., whereas, without correction for refraction, it should have been (for latitude 30° N.) north of latitude 30° by about the same amount; so that, unless the latitudes of places have changed during the last 4,000 years, the Pyramid of Gizeh was not placed in its position so that it might be on latitude 30° N.

What, then, can have been the object in placing the Pyramid at latitude 29° 58′ 81″?

The following is suggested: It was intended to be placed at the latitude where the length of a unit of longitude was 17·6 inches, so that there were 3,600 inches to a minute of arc.

This gives $17{\cdot}6 \times 60 \times 60 = 1{,}760 \times 36$ inches.
$= 1{,}760$ British yards, or 1 mile.

This position would be at the latitude where there are 88 feet to 1 second of arc; at about latitude 29° 53′—some five minutes further south than the Pyramid is situated.

* The Greek Anaximander, about B.C. 550, is said to have determined the latitude of Sparta. The Greek Hipparcus, about B.C. 150, calculated the inclination of the ecliptic and the equator as 23° 51′; it was at that time 23° 46′. He indicated the position of a place on the earth by means of its latitude and longitude.

It would seem probable that the Egyptians thought that according to their calculations they had placed the Pyramid at a point in latitude N. where the seconds of arc of latitude and longitude were as 6 : 7, and that the great circle was 103·2 feet to a second of arc of latitude, though it actually is 101·67 feet.

## British Weights and Measures derived from Egypt.

1. It has been shown that the *British inch* is nearly identical with the primitive or Pyramid inch, and that it is derived from the palm in turning cylindrical into rectangular measure. There is no record as to its first appearance in Britain, and its earliest use as a building unit, at present known, is in churches of the eleventh century. F. Petrie, in 'Inductive Metrology' (p. 111), says: 'That units used by the Britons should be transmitted to the Anglo-Saxons is far from impossible, as the latter probably imbibed much of the fragmentary civilization that they found in the Romano-Britons.' That the Saxon units should descend to mediæval times is most probable, as the Normans were a ruling, and not a working, class. It is probable, however, that the inch first came to Britain in connection with weights and measures, and not as part of a linear measure.

2. It may be surmised that the *British foot* is simply derived from the British 18-inch cubit, or double cubit of 36 inches (or a yard), in the same manner as the Grecian or Roman foot is derived. The yard cubed is 46,656 cubic inches, and this, added to its half (23,328) amounts to 69,984 inches, just 16 (or $\frac{1}{4000}$) short of 70,000 inches.

3. The *British mile* is $8 \times 220$ yards (or double British cubits) or 8 furlongs, and in this corresponds to the division of the base of the Great Pyramid into 220 double cubits of 8 to a minute of arc of meridian.

There are three ancient units which may be connected with the size of the earth :

| | |
|---|---|
| Inch ... ... | There are about 500,500,000 inches in the rotation axis of the earth. |
| Double cubit ... | There are $360 \times 60 \times 60 \times 60 \times 20\frac{11}{18}$ inches (nearly) in a great circle. |
| Base of Pyramid | There are $360 \times 60 \times 8 \times (220 \times 41\frac{2}{9})$ inches (nearly) in a great circle. |

These cannot all and each have been the standard measure.

It is to be noted also that the length of a minute of arc of longitude at the latitude of the Great Pyramid, is nearly $60 \times 60 \times 17{\cdot}6$ inches $= 1760 \times 36$ inches. None of these cases may be that which has to do with the origin of the mile, but it seems obvious that there is a connection between the subdivisions of our mile and the length of base of the Pyramid.

4. The *British measures of capacity* — quarters, bushels, gallons, and pints—have been shown to be closely coincident with those of Egypt, as $69\frac{1}{2} : 70$.

The pound Avoirdupois is one-tenth of the gallon, and weighs 7,000 grains imperial, just as the hon was one-tenth of the hekt, and weighed 6,953 grains imperial. There are 320 hon to a cubic cubit. If the Egyptian table of measures be examined it will be seen that the original measures were cylinders the lengths of whose sides were expressed in palms —*i.e.*, quarters, bushels, gallons, and pints—whereas in the time of Ahmes (1600 B.C.) the apit had been introduced as a principal measure. It would appear, then, that (as we still retain quarters, bushels and gallons as our principal measures of capacity) these measures were introduced into Britain before the time of Ahmes, unless brought in by the Anglo-Saxons: they have no connection with Roman or Greek measures. Thus it is suggested that there may have been

an advanced condition of civilization in our island at a very early period.

### Imperial Superficial Measure.

The following is a table (VI.) showing the relative values of links and yards in superficial measure :

| | | Links. | Square Links. | Yards. | Square Yards. | Inches. | Remarks. |
|---|---|---|---|---|---|---|---|
| 1 | 1 square rod, pole, or perch ... | $(25)^2$ | 625 | $(5\frac{1}{2})^2$ | $30\frac{1}{4}$ | — | — |
| 16 | 1 square chain | $(100)^2$ | 10,000 | $(22)^2$ | 484 | — | — |
| 40 | 1 square rood | $40 \times (25)^2$ | 25,000 | $40 \times (5\frac{1}{2})^2$ | 1,210 | — | — |
| 160 | 1 square acre | $10 \times (100)^2$ | 100,000 | $10 \times (22)^2$ | 4,840 | — | — |
| 1600 | 10 square acres | $(1000)^2$ | 1,000,000 | $(220)^2$ | 48,400 | — | — |

If we examine this table it will be apparent that the only square quantities are the square rod, the square chain and the 10 acres, and that, taking a yard as a double cubit, we have 220 double cubits (of 36 inches) square in 10 acres, just as there are 220 double cubits (of $41\frac{2}{9}$ inches) square in the base of the Pyramid. We may, then, look upon 10 acres as having been the original British standard of measure for square measure ; it is about three-quarters of the area of the Pyramid base. This British superficial measure has the appearance of being far more ancient than the superficial measures of Greece and Rome that have come down to us.

## Table (VII.) showing Early Egyptian and Modern British Measures.

The difference is as 344 : 345·6.

| Egyptian. | | Hon. | Cubic Palms. | Cubic Inches. | Cylinder Palms. | Cylinder Inches. | Cubes Palms. | Cubes Inches. | Parallelopipedon Inches. | Ancient Grains. Measure of Water. | Ancient Grains. Measure of Barley. | Grains Imperial. Measure of Water. | British Measures. Cubic Inches. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Kat | $\frac{1}{50}$ | — | — | — | — | — | — | — | 120·3 | 86 | 138·0 | — |
| Cubic inch ... | — | — | — | 1 | — | — | — | — | — | 220 | 155·5 | — | 1 |
| | Uten | $\frac{1}{5}$ | — | — | — | — | — | — | — | 1,203 | 860 | — | — |
| | Cubic palm | — | 1 | 25·5 | — | — | 1 | — | — | 5,650 | 4,009 | — | 25·8 |
| Pound Avoirdupois ... | Hon | 1 | 1·07 | 27·3 | — | — | — | 3 | — | 6,015 | 4,296 | 6903·1 | 27·7 |
| Egyptian : | | | | | | | | | | | | | |
| Pint ... | — | $1\frac{1}{4}$ | 1·34 | 34·4 | $\frac{3}{4}$ | — | — | $3\frac{1}{4}$ | — | 7,518 | — | — | 34·6 |
| Quart ... | — | $2\frac{1}{2}$ | 2·69 | 68·8 | — | — | — | — | — | — | — | — | — |
| Pottle ... | — | 5 | 5·36 | 136·7 | — | $3\frac{1}{2}$ | $1\frac{1}{4}$ | — | $6\frac{1}{4}$, $6\frac{1}{4}$, $3\frac{1}{2}$ | — | — | — | — |
| Gallon ... | Hekt | 10 | 10·72 | 273·4 | $1\frac{1}{2}$ | — | — | $6\frac{1}{2}$ | — | 60,150 | 42,968 | 69,031 | 277 |
| Peck ... | — | 20 | 21·14 | 546·8 | — | — | — | — | — | — | — | — | — |
| Dell or Tovit ... | Apit | 40 | 42·87 | 1093·7 | — | 7 | $3\frac{1}{2}$ | — | $12\frac{1}{2}$, $12\frac{1}{2}$, 7 | — | — | — | — |
| Bushel ... | — | 80 | 85·71 | 2187·5 | 3 | — | — | 13 | — | 481,250 | 343,750 | 552,249 | 2218 |
| Strike ... | Ram | 160 | 171·5 | 4375 | — | — | — | — | — | — | — | — | — |
| $\frac{2}{3}$ (Cubit)$^3$ | Khar | 213·3 | — | 8533·3 | — | — | — | 18 | — | — | — | — | — |
| Combe ... | Cubit cubed | 320 | 343 | 8750 | — | 14 | 7 | — | 25, 25, 14 | — | — | — | — |
| Quarter ... | — | 640 | 686 | 17,500 | 6 | — | — | 26 | — | 3,850,000 | 2,750,000 | 4,418,015 | 17,745 |
| Chaldron... | — | 2,560 | 2,744 | 70,000 | — | 28 | 14 | — | 50, 50, 28 | 15,400,000 | 11,000,000 | 17,672,060 | 70,982 |

On examination of this table it will be apparent that the first measures were the cylinders of 6, 3, $1\frac{1}{2}$, and $\frac{3}{4}$ palms a side—*i.e.*, quarters, bushels, gallons, and pints; then come the pottle, apit, and combe.

# CHAPTER VI

## BABYLONIAN AND HEBREW MEASURES

There is no divergence of opinion as to the relative proportions of the Hebrew measures derived from Babylon. They are 720 hon, 180 cab, 30 seah, or 10 bath (or ephah) to a cor (or homer). The difference of opinion is as to their absolute value. I have taken the view of Josephus and early writers that the log is nearly identical with the Greek Xestes and Roman Sextarius, so that there are 30 bath, or 2,160 log to a double cubit cubed.

These measures differ entirely from those of the Egyptian system. They have been evolved at a later period, when the mind of man has become accustomed to squares and cubes and elaborate calculations. They, however, are connected with the Egyptian system through the double cubit cubed of 14 palms (or $41\frac{2}{9}$ inches) a side, on which they are based.

2,560 hon or 2,160 log go to a double cubit cubed. The two systems are placed together for comparison:

Egyptian binary system:

$$\frac{70{,}000}{4\times 8\times 8\times 10}=\frac{70{,}000}{2{,}560}=\underset{\text{Cubic Inches.}}{27{\cdot}3}=\text{a hon.}$$

Babylonian system:

$$\frac{70{,}000}{3\times 10\times 3\times 2\times 3\times 4}=\frac{70{,}000}{2{,}160}=\underset{\text{Cubic Inches.}}{32{\cdot}4}=\text{a log.}$$

At first sight the Babylonian system seems to be of a very arbitrary character, but on examination it will be seen that it is governed by the theory of numbers—*i.e.*, the necessity for adhering to whole numbers or simple or moderate fractions, for the sides of the cubic measures. The sides of the cubes are measured in palms or fractions of palms, and not in inches. It will be noticed that 216 is the product of two cubes, $(2)^3 \times (3)^3$, so that the division of the cube sides was not difficult.

It had been discovered that $(14)^3 : (6\frac{1}{2})^3 :: 10 : 1$ nearly,

and that $(6\frac{1}{2})^3 : (4\frac{1}{2})^3 :: 3 : 1$ nearly ;

Therefore the bath can be derived at once from the double cubit cubed :

$$(14)^3) : (4\tfrac{1}{2})^3 :: 30 : 1.$$

The log can also be derived direct :

$$(6\tfrac{1}{2})^3 : (1\tfrac{1}{12})^3 :: 216 : 1.$$

$$\therefore (14)^3 : (1\tfrac{1}{12})^3 :: 2160 : 1.$$

The proportions of the various measures of capacity are obtained from the Bible, Josephus, and the Talmud.

The bath and ephah (wet and dry measure respectively) are equal in capacity (Ezek. xlv. 10, 11). An ephah = 10 omer (Exod. xvi. 36), and a homer or cor = 10 ephah (Ezek. xlv. 10).

The cor is equal to the homer; it is a measure just about one-third more than our quarter, and is one-third of a double cubit cubed. Cor means *round*, and it is a sphere of exactly 6 palms radius. $\frac{4}{3} \times \frac{19}{6} \times (6)^3 = 912$ cubic palms.

In the account of the food given by Solomon for the workmen of Hiram engaged on the Temple — 20,000 *measures* of wheat, etc.—the word 'cor' is translated in the Authorized Version as 'measure.' Again, Solomon is said to have used from day to day in his house 30 measures (cor) of fine flour and 60 measures (cor) of meal (1 Kings iv. 22).

A cor is about 960 lb., so that his provision of meal and flour each day was $90 \times 960$ lb., or sufficient for about 90,000 persons.

The seah is the common household measure for corn, etc., equal to about 3 gallons. It is also translated in the Authorized Version as 'measure.' According to the Rabbins, it is the third part of an ephah or bath. Sarah made cakes of or from 3 measures (an ephah) of fine flour (Gen. xvii. 6). Abigail took 5 measures of parched corn (1 Sam. xxv. 18).

It will be seen that no one measure in this system approaches nearly to one of the Egyptian system—*i.e.*, bath, seah, hin, omer, cab, and log have nothing in common with quarters, bushels, gallons, and hon, except that they are subdivisions of the double cubit cubed of 14 palms a side.

If we test these measurements with the records of the past we shall find that Josephus in six different passages gives Hebrew measures in equivalent Roman measures (see Chapter VIII., Grecian and Roman Measures).

In the 'Encyclopædia Britannica' there are various estimates of the content of the Hebrew bath, the average being 2,304 cubic inches, and the extremes 2,200 and 2,500 cubic inches, the content here given (Table VIII.) being $2333 \cdot \dot{3}$ cubic inches. It is also stated that the Rabbins said that a cubic cubit of 21·5 inches gave a content of 320 logs; this, again, is a close approximation, $(21 \cdot 5)\sqrt[3]{320} = 31 \cdot 65$ cubic inches to a log, instead of 32·4.

There are two passages in the Old Testament which may serve to test the standard of capacity here given, viz., the accounts given of the dimensions of the ten lavers and of the Brazen Sea on the building of Solomon's Temple.

(*a*) In the first case, as the laver was one of the smaller accessories of the Temple, it is assumed that it was measured by the smaller cubit of 6 palms, according to the statement in the Talmud (Menachoth, 97, *a*, *b*; Midd., iii. 1) that the

smaller cubit was used for a portion of the altar, for the altar of incense, and for the utensils of the Temple.

The diameter given is 4 cubits (1 Kings vii. 26), and, according to the description in the Bible, Septuagint, and Josephus, the shape was hemispherical. We know that a cylinder of 6 palms height and radius contains 17,500 cubic inches. Therefore this hemisphere of 12 palms radius will measure $\frac{2 \times 8 \times 17{,}500}{3} = 93{,}333{\cdot}\dot{3}$ cubic inches, and as they are equivalent to 40 bath, the bath should measure $2{,}333{\cdot}\dot{3}$ cubic inches, as given on the preceding page. There is thus entire accord in these measurements.

(*b*) As regards the Brazen Sea, it can be seen at a glance that there is a discrepancy in the measures, as a hemisphere of 3,000 baths would be 75 times the content of a laver, and therefore would be more than 16 cubits in diameter, instead of 10 cubits (1 Kings vii. 26, and 2 Chron. iv. 5). Moreover, the two accounts give different volumes (2,000 and 3,000 baths). A hemisphere 5 cubits of 7 palms in radius would hold $\left(\frac{35}{12}\right)^3$ the capacity of the laver; this works out to 24·8, so that the Brazen Sea holds almost exactly 992 baths. It seems probable, then, that the correct reading of the 2,000 and 3,000 baths (1 Kings vii. 26 and 2 Chron. iv. 5) should be 1,000 baths.

It follows from this that a bath can also be in the form of a hemisphere diameter $\frac{1}{10}$ diameter of Great Sea—*i.e.*, 1 cubit of 7 palms. This is the only one of the Hebrew measures which can be constructed in form of a hemisphere, using whole numbers for palms. The bath can also be in the form of a cylinder of 9 inches height and radius.

The Hebrew cor (= 10 baths) is exactly half an English cubic yard. If the contents of a cubic yard are shaped in the form of a sphere and a cylinder enclose it, and a cone

be erected on the base of the cylinder with the same height, the cone will represent exactly a cor, the sphere will represent a cubic yard, and the cylinder will represent an Egyptian chaldron or 4 quarters, and their proportions respectively will be 1, 2, and 3.

## Hebrew Square Measure.

The cubes forming the measures of capacity are 14, $6\frac{1}{2}$, $4\frac{1}{2}$, $3\frac{1}{8}$, $2\frac{12}{25}$, $2\frac{1}{10}$, $1\frac{18}{25}$, $1\frac{1}{12}$ palms a side. There is only one of these which can directly be adapted to square measure—viz., $3\frac{1}{8}$ cubic palms, the value of the seah—and therefore the ancients have taken it as the standard.

The theory of the Hebrew square measure, as being derived from cubic measure, is that, given a certain number of grains, and placing them a certain distance apart, the cubic measure should hold sufficient corn-grains to sow over the square measure. In other words, a seah of land is the area that can be sown with a seah of corn.

We know from records of the past ('Encyclopædia Britannica': 'W. and M.') that the seah of square measure was 50 cubits square.

The object now is to ascertain how one was derived from the other.

We must first have some notion as to the number of ancient grains of barley contained in a log, cab, seah, etc. Of course the number may be a purely conventional one. We know that a log of water is of 32·4 + cubic inches capacity, and weighs 8,181·5 imperial grains. (See Table VIII.)

Then the question is, What may be the weight of the log when filled with barleycorns?

In Piazzi Smyth's list of specific gravities ('Our Inheritance in the Great Pyramid') he gives barley (*loose* as in bushel) : distilled water : : 112 : 175. Whitaker's Almanack gives

## Table (VIII.) of Babylonian and Hebrew Measures.

| Log. | Cubes. | | Name. | | Cubic Palms. | Cubic Inches. | Ancient Grains. | | Imperial Grains. |
|---|---|---|---|---|---|---|---|---|---|
| | Side in Palms. Decimals. | Side in Palms. | Hebrew. | Babylonian. | | | Weight of Water. | Grain in Bulk. | |
| — | $\frac{1}{3000}$ talent | — | Shekel | — | — | — | 192 | 133·8 | 218·3 |
| — | — | — | Cubic inch | — | — | 1·0 | 220·0 | 157·1 | 252·458 |
| — | — | — | Cubic palm | — | 1·0 | 25·5 | 5,610 | 4,009 | 6,476 |
| 1 | 1·0830 | $1\frac{1}{12}$ | Log | — | 1·270 | 32·4 | 7,129<br>7,145<br>7,200 | 5,092<br>5,104 | 8,181·5 |
| 4 | 1·7192 | $1\frac{18}{25}$ | Cab | Ka | 5·081 | 129·6 | 28,800 | 20,416 | — |
| 7·2 | 2·0193 | $2\frac{1}{10}$ | Omer | — | 9·261 | $233\cdot\dot{3}$ | 5,840 | 36,750 | — |
| 12 | 2·4795 | $2\frac{12}{25}$ | Hin | — | 15·23 | $388\cdot\dot{8}$ | 86,400 | 61,250 | — |
| 24 | 3·1242 | $3\frac{1}{8}$ | Seah | Bar | 30·46 | $777\cdot\dot{7}$ | 172,000 | 122,500 | — |
| 72 | 4·5056 | $4\frac{1}{2}$ | Bath | As | 91·4 | $2{,}333\cdot\dot{3}$ | 518,400 | 367,500 | — |
| 80 | $4\cdot\dot{6}$ | $4\frac{2}{3}$ | Talent | — | 101·5 | 2,592·4 | 576,000 | 411,400 | 654,520 |
| 216 | 6·49822 | $6\frac{1}{2}$ | — | — | 274·4 | 7,000·0 | 1,555,200 | 1,102,500 | — |
| 720 | 9·7069 | $9\frac{7}{10}$ | Cor | Gur | 914·6 | $23{,}\dot{3}33\cdot\dot{3}$ | 5,184,000 | 3,675,000 | — |
| 2160 | 14·0 | 14 | 4 quarters | — | 2744·0 | 70,000·0 | 15,400,000<br>15,552,000 | 11,000,000<br>11,025,000 | 17,672,064 |
| 2880 | Hemisphere | — | Laver | — | 3658·4 | 93,333·3 | — | — | — |

The talent or shekel are placed in the table for convenience of reference; they are deduced in Chapter VII.

a bushel of water (80 lb.) to the following: Mediterranean barley, 50 lb.; French barley, $52\frac{1}{2}$ lb.; English barley, 56 lb., giving a proportion:

$$\begin{array}{lcl} 50 & : & 80 \\ 52\frac{1}{2} & : & 80 \\ 56 & : & 80 \end{array}$$

But this again appears to be for loose barley. The measure of barley of ancient times was full measure, running over, pressed down, so that there was a considerable percentage more than merely loose barley.

Heaped measure was abolished in this country in 1878 (Chaney, 'W. and M.,' p. 128), but yet the grain is not now supposed to be put in *loosely*, but shaken down from a height of 2 to 3 feet, and *pressed* and struck, giving an addition of about 13 per cent. to the loose measure already indicated. By adding on this 13 per cent. we get the following averages:

| | Barley. | Water. |
|---|---|---|
| Piazzi Smyth ... ... | $126\frac{1}{2}$ | 175 |
| Mediterranean barley ... | $123\frac{1}{2}$ | |
| French barley ... ... | $130\frac{5}{10}$ | |
| English barley ... ... | 137 | |

We will take a low average of 125, giving a proportion of water to barley as 7 : 5, and applying this to the log we get the weight of barley grains in it as:

$$\frac{8{,}181 \cdot 5 \times 5}{7} = 5{,}844 \text{ imperial grains.}$$

That is to say, a log of barley weighs about 5,844 imperial grains as compared with a log of water. This is merely a tentative inquiry to ascertain approximately the number of ancient barley grains to a log. It is not to be supposed that imperial grains are equal to ancient barley grains; judging by what

takes place usually, there should be a considerable amount of degradation in the imperial grain, and we may probably assume that the number of barleycorns in a log was nearer to 5,000 than to 5,891.

Assuming, then, that the number is somewhere about 5,000, we can now ascertain conventionally the area of land that a seah will sow.

The distance apart of each grain must be deduced. We will therefore take the following distances for trial: 2, 3, 4, 5 digits.

There are 2,500 square cubits in a square seah. This would give the following number of grains at the numbers 2, 3, 4, 5 digits apart, viz.:

| | |
|---|---|
| Cubit of 6 spans ... ... | None suitable. |
| Cubit of 7 spans ... ... ... | 61,250·0 |
| | 91,875·0 |
| | 122,500·0 |
| | 153,025·0 |

Out of these, the only one suitable is the distance of 4 digits or 1 palm, giving 122,500 grains. There are 24 logs to a seah; this gives 5,104 barley grains to a log. An inspection of these numbers will show that the cubit is one of 7 palms.

The only symmetrical method of arranging 122,500 conventional barleycorns in a seah so as to preserve whole numbers is for the base to number $50 \times 50$ grains, and the height 49 grains. By this artifice cubic measure may be turned into square measure.

The base of the seah measures $\frac{25}{8}$ palms square. There will be 49 of these layers, and if all are laid in one layer the square surface will be $\frac{25}{8} \times 7$ palms a side.

Multiply by 16 each way, and we get a square area of $50 \times 7$ palms a side, or 50 cubits square of 7 palms each, giving a distance from centre to centre of barleycorns of 1 palm.

This seems to be a small number of grains for sowing such an area of ground (about half what is used in England at the present day), but I think there can be little doubt it was the conventional number which connected square with cubic measure.

Note that each conventional grain is a *cube* occupying the space of $\frac{1}{16}$ square palm a side.

It is suggested that the 500 cubits square given by the Talmudists as the area of the Temple enclosure was derived from the area circle of the Pyramid base, and was 500 cubits of $20{\cdot}61\dot{0}\dot{9}$ inches.

We shall now be able to apply a test to the number of grains found to a log by examination of the Babylonian weights, viz.: Given 5,104 barley grains to a log, what is the weight of the log of water in terms of these grains?

$\frac{5{,}104 \times 7}{5} = 7{,}145$ grains.

At 2,160 logs to a double cubed cubit, it gives 15,433,200 ancient grains to the double cubit cubed, as against 15,400,000 ancient grains obtained when considering Egyptian measure. Or, taking it by bulk at 122,500 grains to a seah and 90 seahs to a double cubit cubed, it gives 11,025,000 grains to a double cubit cubed as against 11,000,000 obtained from considering Egyptian measure.

If we look upon the matter as we did in Egyptian measure we should suppose that there were an even number of grains in a log (a multiple of 6), and Madden ('Jewish Coins,' pp. 267, 289) arrived at the conclusion that the number was 7,200.

Again, if we divide 154,000,000 by 2,160 we get 7,129 ancient grains to a log.

We have thus the following discrepancies :

7,129 ancient grains to a log as compared with Egyptian measure, *i.e.*, $\frac{15,400,000}{2,160} = 7,129.$

7,145 ancient grains to a log, derived from turning the cubic seah into a seah of land.

7,200 ancient grains to a log, considered as a probable round number likely to be used.

Now, we have not to consider which is rigidly accurate, but which the ancients used. We have the proportion of 7 to 5 as a purely conventional ratio (though very near the mark) for the relative weights of water and barley, and again we have differences in the weights of a cubic inch of rain-water and river-water. The difference is about 1 per cent., and it seems probable that the Babylonians and Hebrews adopted the number of 7,200 ancient grains to a log, although it led to their grain differing by 1 per cent. from the older Egyptian grain.

It must be recollected that the early Egyptian (or so-called Egyptian) system, with its weights, was probably in use some hundreds of years before the cubic measure of the Babylonians and Hebrews was introduced.*

First we have 4,000 ancient grains in bulk to a cubic palm ; then we have 6,000 grains weight of water to a hon, giving 220 ancient grains to a cubic inch ; and then we have subsequently the introduction of the log, which rigidly correct in weight should be 7,129 grains, but which probably was considered to weigh 7,200 grains.

The remarkable manner in which these various measures divide into the double cubit cubed can now be pointed out :

* On the assumption that the cylindrical measures of Egypt were the original measures of the ancients before they drifted apart. (See Chapter III.)

| | | Ancient Grains. |
|---|---|---|
| 70,0000 cubic inches of 220 grains each | | = 15,400,000 |
| $(14)^3$ = 2,744 ,, palms of 5,600 | ,, | = 15,366,400 |
| $(4)^3 \times 10 = 2{,}560$ hon of 6,000 | ,, | = 15,360,000 |
| $(6)^3 \times 10 = 2{,}160$ log of 7,200 | ,, | = 15,552,000 |
| (Land measure) 2,160 log of 7,145 | ,, | = 15,433,200 |

## CHAPTER VII

### THE BABYLONIAN TALENT

It is accepted by modern writers that the standard Babylonian talent for silver and merchandise was represented by the weight of a Babylonian cubic foot of rain-water—the foot being two-thirds of the cubit—and that this talent was divided into 80 units or pounds for commercial purposes.

Hussey ('Ancient Weights and Measures') mentions that:

Varro states that an Egyptian talent was equal to 80 Roman pounds.

Polybius states that an Euboic talent was equal to 80 Roman pounds.

Livy states that an Attic talent was equal to 80 Roman pounds.

Böckh (as quoted by Grote in 'Class. Mus.,' vol. i., p. 4) states that the Babylonian cubic foot weighed 60 Eginetan minæ (= 1 Babylonian talent).

It may therefore be assumed that the ancient standard talent for silver and merchandise was the weight of two-thirds the ancient cubit ($20{\cdot}6\dot{1}0\dot{9}$ inches) cubed of rain-water, viz., a cube of $4\frac{2}{3}$ palms, or 13·7406 inches a side.

$$\frac{70{,}000}{27} = 2592{\cdot}4 \text{ cubic inches} = 654{,}520 \text{ imperial grains.}$$

This ancient foot of 13·7406 inches does not seem to have been used in early days as a standard of linear measure, but only in connection with the standard weight of the talent.

It was a simple matter to divide the cubic foot into eighty parts.

It can be shown that $(14)^3 : (6\frac{1}{2})^3 :: 10 : 1$,
and $(4\frac{2}{3} \times 3) = 14$;

$$\therefore (4\frac{2}{3}) : (6\frac{1}{2} \times \frac{1}{6}) :: 80 : 1;$$

$\therefore$ The measure of the cubic log is $1\frac{1}{12}$ palms a side.

The Babylonian talent appears to have been also divided for monetary purposes into 60 minæ, but the Hebrew talent is variously computed as divided into 50 or 60 minæ, probably according to whether it was of gold or silver, the talent of gold being divided into 50 minæ and being by weight a twelfth of the talent of silver.

We may, then, consider the ancient talent of silver of 654,520 imperial grains to have been divided as follows:

| Talent of Silver. | Mina. | | Proportions. |
|---|---|---|---|
| | Imperial Grains. | Ancient Grains. | |
| For ordinary weights, 80 minæ | 8,181·5* | 7,200 | 15 |
| For silver ... ... 60 „ | 10,908 | 9,600 | 20 |

giving respectively to the minæ proportions of 15 and 20.

The following relative proportions of the Grecian talents are given by ancient writers, and accepted in the articles in Smith's 'Biblical Dictionary' and 'Dictionary of Greek and Roman Antiquities':

1. That the old Macedonian, Eginetan, or Babylonian talent bore to the Euboic or old Attic talent the proportion of 6 : 5, and to the Solonian or newer Attic talent the proportion of 5 : 3.

* This is the Hebrew log, and corresponds to the pound in the Grecian and Roman systems. As will be seen subsequently, the early Grecian pound is two-thirds 8,181·5, or 5,454·3 imperial grains.

2. The Euboic talent thus bore to the Solonian the proportion of 100 : 72, or 25 : 18.

Authorities also agree that there were 60 minæ, 3,000 shekels, or 6,000 drachmæ to a silver talent. We shall thus have the following proportions :*

TABLE IX.

| | Imperial Grains. | | | Ancient Grains. | | | Proportions. |
|---|---|---|---|---|---|---|---|
| | Talent or Cubic Foot | Mina. | | Talent or Cubic Foot | Mina. | | |
| Babylonian | 654,480† | 10,908 | Shekel. 218·1 | 576,000 | 9,600 | Shekel. 192 | 30 |
| Eginetan ... | 654,480 | 10,908 | Drachma. 109·08 | 576,000 | 9,600 | Drachma. 96 | 30 |
| Old Attic, or Euboic | 545,500 | 9,090 | 90·9 | 480,000 | 8,000 | 80 | 25 |
| Newer Attic, or Solonian | 392,760 | 6,546 | 65·5 | 345,600 | 5,760 | 57·6 | 18 |

It will thus appear that the ancient silver talent contained 3,000 shekels of 218·1 imperial grains each, while the Eginetan talent contained 6,000 drachmæ of 109·08 imperial grains each. We can now ascertain how far the weights of existing coins are in accordance with these deductions.

Hussey ('Ancient Weights and Money'), after many trials,

* From these proportions it will be seen that the Eginetan talent contained 10,000 Solonian or Attic drachmæ, as stated by Pollux (ix. 76, 86).

† I have taken a round number, 654,480 imperial grains, in lieu of 654,520 imperial grains (p. 79), because it admits of division by 60 and 80.

gives the silver shekel of Simon Maccabæus as weighing 218 imperial grains, or about half a Roman ounce, and cites Arius Montanus, Villalpandus, Greaves, Mersennus, and Eisenschmid as giving the weight of the shekel as from 218 to 220 grains, and Barthelmy as giving it as 217 grains. My deduction of 218·1 grains thus accords with these practical trials.

Madden ('Jewish Coinage,' p. 281) considers 220 to be about the average weight of the Maccabæan shekel, and therefore makes the weight of the silver talent some 5,000 grains heavier (660,000 instead of 654,520) than the weight deduced by me ; difference, about 1 per cent.

As to whether the shekel of the Maccabees was the same weight as that in use before the captivity we have no means of ascertaining. The Talmudists seem to think it was increased in weight subsequently, probably doubled in weight, as will be shown.

Hussey gives 66·5 imperial grains (from actual trials) for the newer Attic drachma, and Greaves gives 67 imperial grains, while the weight deduced from the ancient talent and proportion of 5 : 3 is 65·5 grains.

No drachmæ of the Euboic standard appear to have been examined, but Hussey's deduced value is 92 grains and Böckh's value is 93½ grains, while the weight now deduced is 91 grains. There appear to have been no comprehensive trials of the Eginetan coins except by Hussey, who made the weight of a drachma but 96 grains, whereas it should be theoretically 109 grains; but the writer of the article 'Nummus' in Smith's 'Dictionary of Greek and Roman Antiquities' states that there are coins of the Eginetan system which come up nearly to the full theoretical weight.*

The table of weights from Nineveh (Layard's 'Nineveh and Babylon'), some of which are in good preservation, gives

* Namely, those of Melos and Byzantium, both Dorian settlements, and those of the Macedonian Kings before Alexander the Great.

several tests of the weight of the mina, and Madden considers that one of 15,984 imperial grains is up to full weight, and represents two minæ of 7,992 grains. The weight deduced from the ancient cubit is 8,181, giving a difference of 188 grains, or nearly 2 per cent. (p. 80). This may be owing to a depreciated cubit having been in use at this comparatively late period (700 to 800 years B.C.), or else to the weight not being up to the full standard.

Madden (p. 267) allows 60 of these weights to a talent, and thus deduced a *lighter* Babylonian talent of 479,520 imperial grains. I venture to think that these weights belong to the commercial weights of Babylon, 80 to a talent, and that they simply form portions of the talent of 654,600 imperial grains.

Madden (p. 268) further, by taking 67·5 grains to an Attic drachma, and 72 Attic minæ to the Babylonian talent, arrives at a light Babylonian talent of 486,000 imperial grains. It is, however, pointed out by the writer of 'Nummus' (Smith's 'Dictionary of Greek and Roman Antiquities') that it was the *old* Attic or Euboic minæ, of which there were 72 to a Babylonian talent. The weight of this mina was about 9,097·2 imperial grains, which gives 655,000 imperial grains to a Babylonian talent. In consequence of the introduction of a lesser Babylonian talent by Madden, there is great discrepancy in the estimates as to the Grecian talents given in Smith's 'Greek and Roman Antiquities' and in Smith's 'Biblical Dictionary.' I have no doubt that the former is more correct.

## *Ancient Authorities.*

Epiphanius estimates the Hebrew silver talent at 125 Roman pounds (Roman pound taken at 4,988 and 5,235 grains imperial):

6,200 × 105 = 651,000 grains to a talent.
654,480 " (p. 79).

3,480 Difference, ½ per cent.

6,236 × 105 = 654,800
654,480 (p. 79).

320 Difference 1 in 2,000

Plutarch (Solon 15), on testimony of Androtion: 'Solon made the mina of 100 drachmæ, which had formerly contained 73.'

It is shown in article 'Pondera' (Smith's 'Dictionary of Greek and Roman Antiquities') that Plutarch, by neglecting fractions in his computations, had probably altered the original ratio of 100 : 72 into 100 : 73.

Herodotus says Babylonian talent = 70 Euboic minæ (iii. 89).

At 6 : 5 it really equals 72 Euboic minæ.

Pollux says the Babylonian talent contains 70 [old] Attic minæ and 7,000 Attic drachmæ (ix. 86).

Ælian (loc. cit.) said that the Babylonian talent contains 72 [old] Attic minæ.

| | | | | |
|---|---|---|---|---|
| Old Attic or Euboic talent | = | 8,333⅓ | Solonian | drachmæ. |
| Euboic mina ... ... | = | 138 8/9 | " | " |
| (138 8/9 × 72) ... ... | = | 10,000 | " | " |

## CHAPTER VIII

### SOLONIAN, GRECIAN, AND ROMAN MEASURES

It has been conjectured that the later Grecian and Roman measures of capacity have been derived from those of Babylon, because they are nearly equal to them, and follow the same system of 60 units (pounds or minæ) in a cubic foot, while under the Egyptian system it is the bushel and not the cubic foot that is divided into 60 units.

It will be found on examination that also the Solonian measures of capacity are not only on exactly the same system, but are precisely the same as those of Babylon, with the exception that the pound has been altered—i.e., the measures of capacity have remained the same, while the weights have been altered. In this respect they are on all fours with the British measures of capacity. As the British measures have retained the Egyptian measures of capacity and the hen, or pound, Egyptian, and taken another pound (the Tower pound), so the Solonian measures have retained the Babylonian measures with the log or pound (i.e., the Sextarius), and have also taken another pound one-eightieth of the cubic foot. (See Tables VIII. and XI.)

The following comparison shows that the Solonian, Grecian, and Roman systems are identical in proportions with the Babylonian and Hebrew measures:

| Babylonian and Hebrew. | Log. | Solonian, Grecian, and Roman. | |
|---|---|---|---|
| | | Pound ... | $\frac{1}{80}$ of cubic foot |
| Log or mina ... ... | 1 | Sextarius ... ... ... | 1 |
| Cab ... ... ... | 4 | Congius ... ... ... | 6 |
| Hin ... ... ... | 12 | Modius ... ... ... | 16 |
| Seah ... ... ... | 24 | Urna ... ... ... | 24 |
| 2 seahs ... ... ... | 48 | Amphora or cubic foot ... | 48 |
| Ephah or bath ... ... | 72 | Metrêtês ... ... ... | 72 |

Thus, the log, seah, and bath correspond in proportions respectively to the sextarius, urna and metrêtês, and the cab and hin are related to the congius and modius.

The $\left(\frac{2}{3}\text{ cubit}\right)$ cubed or cubic foot of Greece, Rome, and of Europe generally is about two-thirds the capacity of that of Babylon, and the question arises whether this is due to the gradual depreciation of the cubit of $20{\cdot}6\dot{1}0\dot{9}$ inches to about 18 inches, or to a deliberate change of unit for purposes of convenience.

No doubt the old system of working in palms had many inconveniences, and the new unit of 70,000 to a double cubit cubed could not be fairly brought into use generally so long as the standard of length remained a broken number of inches —viz., $20{\cdot}6\dot{1}0\dot{9}$. The change that was made was comparatively a simple one, by which a new pound (80 to a new cubic foot) became half the monetary mina, of which there were 60 to an ancient cubic foot or talent, and by which the standard cubit of 20·6109 inches became 18 inches and the double cubit became a yard, or 36 inches.

This change consisted in reducing the content of the double cubit cubed by one-third, and using the side cube of this remaining bulk as the new standard of length.

70,000 is nearly exactly $\frac{3}{2}\,(36)^3$.

$$(36)^3 = 46{,}656\cdot 0$$
$$\tfrac{1}{2}(36)^3 = 23{,}328\cdot 0$$
$$\overline{69{,}984\cdot 0}$$

Difference, 16 in 70,000, or $\frac{1}{5000}$.

The new double cubit, therefore, now measured 36 inches and the new foot 12 inches; the new pound would therefore weigh 21·6 cubic inches of rain-water, or about 5,454 grains imperial. When corrected for depreciation it is 5,400 grains imperial: the Tower pound (see Chapter XI.).*

The result of this change is tabulated below. The change is likely to have taken place before 700 B.C.:

TABLE X.—BABYLONIAN AND $\frac{2}{3}$ BABYLONIAN (BRITISH) SYSTEMS.

| Double Cubit. Cubed Cubic Inches. | Inches. | | Talent or $\frac{2}{3}$ Cubit Cubed. C.I. | C.I. in Mina or Pound. | Imperial Grains. | Ancient Grains. | |
|---|---|---|---|---|---|---|---|
| | Cubit. | $\frac{2}{3}$ Cubit or Foot. | | | | | |
| 70,000 | 20·6109 | 13·7406 | 2,592 | 32·4 | 8,181 | 7,200 | Commercial mina. |
| | | | | 43·2 | 10,908 | 9,600 | Silver mina. |
| 46,656 | 18·0 | 12·0 | 1,728 | 21·6 | 5,454 | 4,800 | Tower Pound. |

It was M. Böckh ('Metrol. Untersuch.') who first called attention to this ratio in saying 'that the Babylonian cubic foot, standing as it does in the ratio of 3 : 2 to the Grecian cubic foot, weighs 60 Eginetan minæ (= 60 Babylonian minæ = 1 Babylonian talent) of rain-water' ('Class. Mus.,' vol. i., p. 4). To this Mr. Grote in his review of Böckh's work objects that 'his proofs of the ratio of 3 : 2 between the Babylonian and the Grecian foot will be found altogether defective.' It may have been defective in application owing

* The Tower pound is to the Troy pound as 15 : 16.

to an incorrect length of the ancient cubit having been taken, but yet it was right in principle. The application, however, is to the British, and not to the later Grecian or Olympic, foot as now accepted; from which it would appear that this ancient foot is now represented correctly by the British foot, but that in Greece and Italy and other countries it became vitiated or has disappeared.

I call this talent 'the $\frac{2}{3}$ Babylonian (Grecian and British),' and the foot derived from it 'the reduced Babylonian, or British foot.'

M. Böckh also proposed a ratio of 10 : 9 between the Eginetan and Roman pound, which Grote considers altogether inadmissible, and even denies that (properly speaking) there is any such thing as an Eginetan pound, or that there is any fixed normal relation between Grecian weights and Grecian measures, either of length or capacity.

According to the testimony of ancient writers, the Grecian or Olympic foot bears to the Etrurio Roman foot the proportion of 25 to 24. I have taken the Olympic foot at 12·137 inches, giving a cubic foot of 1,800 cubic inches; thus containing 60 minas on the Gudean system (p. 113). Smith's 'Dictionary' gives 12·135 for length of this foot. The Etrurio-Roman foot I have taken at 11·657 inches, giving 5000 grains imperial to the Roman pound (see pp. 90, 93).

Now the various proportions of the talents were:

Babylonian : Newer Attic or Solonian : : 30 : 18.
Babylonian : $\frac{2}{3}$ Babylonian : : 30 : 20.
∴ $\frac{2}{3}$ Babylonian : Solonian : : 20 : 18 : : 10 : 9.

Hitherto it has been a question how the Roman standard has been arrived at. It is suggested that the early Roman coincided with the Solonian system, and we arrive at the following deductions:

The Roman amphora quadrantal, or cubic foot = 48 sextarii. But the Roman, *i.e.* Solonian, cubic foot is:

$$\left(\frac{2}{3}\times\frac{9}{10}=\right)\frac{3}{5}\text{ Babylonian}=80\text{ logs}\times\frac{3}{5}.$$

$\therefore$ The early Roman or Solonian cubic foot = 48 logs.
$\therefore$ the early sextarius = the log.

Also the metrêtês = $1\frac{1}{2}$ amphora = 72 logs.
But the bath = 72 logs.
$\therefore$ The metrêtês = the bath.

It follows that the seah must equal the urna, and the hin equal 2 Attic choes.

The kor also would equal 10 metrêtæ.

This is all in accordance with the statements of Josephus, as follow:

| | | | |
|---|---|---|---|
| 'Ant.,' viii. 2, 9 | ... | Bath | = 72 xestæ. |
| | | $\therefore$ Bath | = metrêtês. |
| 'Ant.,' iii. 8, 3, and 9, 4 | | Hin | = 2 Attic choes. |
| 'Ant.,' ix. 4, 5 ... | ... | Seah | = $1\frac{1}{2}$ Italian modii. |
| | | | = Urna. |
| 'Ant., xv. 9, 2... | ... | Kor | = 10 Attic medimni. (Metrêtês taken.) |
| 'Ant.,' iii. 6, 6 | ... | Omer | = 7 Attic cotylæ. (Xestes taken.) |

Thus, the Solonian and Early Roman measures of capacity are identical with those of Babylon, while the Grecian and Roman (according to Smith's 'Dictionary of Greek and Roman Antiquities') are larger by about 2 per cent. The cubic foot and derived pounds in each case differ according to the length of the foot. (See Tables VIII., XI., and XIII.)

TABLE (XI.) SHOWING THAT THE LATER GRECIAN AND ROMAN MEASURES OF CAPACITY WERE THE SAME AS THOSE OF THE BABYLONIAN AND HEBREW TO WITHIN 3 PER CENT., BUT THAT THE POUND WEIGHT HAD UNDERGONE A CHANGE.

| Grecian Measures. | Roman Measures. | Sextarius. | Late Grecian and Roman Measures, from Smith's 'Dictionary of Greek and Roman Antiquities.' | Solonian Measures (see Table VIII., pp. 73-86). | Hebrew Measures corresponding. |
|---|---|---|---|---|---|
| | | | Cubic In. | Cubic In. | |
| — | Pound $\frac{1}{80}$ cubic foot | (Calculated) | 19·8 | 19·4 | |
| Xestes ... | Sextarius | 1 | 33·28 | 32·4 | Log. |
| Choes ... | Congius | 6 | 199·67 | 194·0 | ½ hin. |
| Hemekton | Semimodius | 8 | 266·24 | — | 8 logs. |
| Ektos ... | Modius | 16 | 532·48 | — | 16 logs. |
| — | Urna | 24 | 798·53 | 777·7 | Seah. |
| — | Amphora quad., or | 48 | 1,597·06 | 1555·5 | |
| | Roman foot cube | (Calculated) | 1,584·0 | Solonian cubic foot. | |
| Metrêtês ... | Mêtrêtes | 72 | 2,396·1 | 2,333·3 | Bath. |
| Medimnos | Medimnus | 96 | 3,194·12 | 3,111·1 | 4 seahs. |
| — | Culeus | 960 | 31,941·2 | 31,111·1 | 40 seahs. |
| — | 1 cubic yard (British) | Nearly. 1,440 | 46,656 | 46,666·6 | 2 kors. |
| — | 4 quarters (Egyptian) | 2,160 | 70,000·0 | 70,000·0 | 3 kors. |

Smith's Dictionary gives both 5,053·2 and 4,988 imperial grains as the weight of the Roman pound. The latter is at the rate of 80 pounds to a cubic foot (11·65 inches) at 252·5 imperial grains to a cubic foot of rain-water. This pound is derived from the Etrurio Roman foot and is Akkadian in its origin.

There is another Roman pound which is nearly equal in weight to the modern Roman pound of 5,234 imperial grains. The principal estimates are as follows. Arbuthnot ('Ancient Coins'), 5,346 imperial grains; Greaves ('The Roman Foot'), 5,256 imperial grains; Hussey, 5,204 imperial grains. This pound is $\frac{14}{9}$ of the Solonian pound.

TABLE XII.—COMPARISON OF WEIGHTS OF TALENTS GIVEN BY VARIOUS AUTHORITIES IN IMPERIAL GRAINS.

| | Name of Talent. | Now Deduced, 1899-1902. | Smith's 'Dictionary of Greek and Roman Antiquities.' | Smith's 'Dictionary of the Bible.' | Conder, 1902. | Hastings' 'Dictionary of Bible,' 1902. |
|---|---|---|---|---|---|---|
| 30 | Babylonian and Hebrew ... | 654,600 | 665,000 | 660,000 | 960,000<br>480,000 | 758,000 gold<br>673,000 silver |
| $\frac{810}{32}$ | Egyptian (Bushel) | 555,050 | — | — | — | — |
| 25 | Euboic ... | 545,500 | 553,000 | 558,880 | 400,000 | — |
| 20 | $\frac{2}{3}$ Babylonian (Grecian and British) | 436,320 | — | — | — | — |
| 18 | Solonian and Early Roman | 392,760 | 414,000 | 405,300 | 400,000 | — |
| — | Late Roman ... | 399,000 | 394,439 | — | — | — |

The dates of the various standards of measure may be surmised:

Eginetan, Macedonian } received about B.C. 750 from { Babylonia.(?) Assyria.(?)

Old Attic, Euboic } received about B.C. 600 from Egypt.

$\frac{2}{3}$ Babylonian (Grecian and British), nothing known. Probably from Assyria.

Solonian, initiated in Greece B.C. 594.

Early Roman, Solonian from Sicily(?) B.C. 300.

Olympic, B.C. 500. From Babylonia.

Etrurio Roman from the Etruscans (Akkadian) B.C. 1,000.

Late Roman, time of Roman Emperors, $\frac{16}{15}$ of Solonian.

In Table XIII. the comparison of the various talents with weights of minæ and pounds is given. The mina is that for silver and the pound is the commercial mina for ordinary weights, 80 to a cubic foot. They bear the proportion of 4 : 3. The Egyptian talent is the bushel (not the cubic foot), a cube of 13 inches a side.

## Table XIII.—Comparison of Weights of Silver Talents.

| Name of Talent. | Ratio of Content. | Cubit. | Foot. | Cubic Foot. | Imperial Grains. | | | Cubic Inches. | | Ancient Grains. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Inches Linear. | Inches Linear. | C.I. | Talent or Cubic Foot. | Silver Mina. $\frac{1}{60}$ | Pound. $\frac{1}{80}$ | Mina. $\frac{1}{60}$ | Pound. $\frac{1}{80}$ | Talent or Cubic Foot. | Mina. $\frac{1}{60}$ | Pound. $\frac{1}{80}$ |
| Babylonian ... | 30 | $20{\cdot}61\dot{0}\dot{9}$ | 13·7406 | 2,592·5 | 654,600 | 10,908 | 8,181 | 43·2 | 32·4 | 576,000 | 9,600 | 7,200 |
| Egyptian (bushel) | $\frac{810}{32}$ | $20{\cdot}61\dot{0}\dot{9}$ | 13·0 | 2,188 | 554,600 | 9,204 | 6,903 | 36·4 | 27·3 | 480,000 | 8,000 | 6,000 |
| Euboic ... | 25 | 19·36 | 12·926 | 2,160 | 545,500 | 9,090 | 6,817 | 36·0 | 27·0 | 480,000 | 8,000 | 6,000 |
| $\frac{2}{3}$ Babylonian (Grecian and British) | 20 | 18·0 | 12·0 | 1,728 | 436,320 | 7,272 | 5,454 | 28·8 | 21·6 | 384,000 | 6,400 | 4,800 |
| Olympic ... | $\frac{125}{6}$ | 18·205 | 12·137 | 1,800 | 454,500 | 7,575 | 5,656 | 30·0 | 22·5 | 400,000<br>396,000 | 6,666<br>6,600 | 5,000<br>4,950 |
| Early Roman<br>Solonian | 18 | 17·3787 | 11·5858 | 1,555·2 | 392,760 | 6,546 | 4,908 | 23·92 | 19·44 | 345,600 | 5,760 | 4,320 |
| Etrurio-Roman (for measures) | $\frac{55}{3}$ | 17·485 | 11·657 | 1,584 | 399,000 | 6,666 | 4,999 | 26·4 | 19·8 | 352,000 | 5,866 | 4,400 |
| Later Roman (for weights) | 19·2 | 17·757 | 11·838 | 1,659·2 | 418,544 | 6,983 | 5,235 | 27·6 | 20·7 | 368,640 | 6,144 | 4,608 |

Note.—There is a discrepancy between the ancient grains Egyptian and Babylonian; taking 6,000 Egyptian to a hon, there should be 7,130 to a log, instead of 7,200. The number of grains ancient to a cubic foot are as follows: Egyptian, $219\frac{3}{7}$; Conventional, 220; Babylonian, $222\frac{2}{9}$; the Egyptian talent is the bushel $\frac{1}{32}$ of the double cubit cubed, while the Babylonian talent or cubic foot is $\frac{1}{27}$ double cubit cubed.

Note.—In Table XIII. the lengths of the Olympic foot and the Etrurio-Roman foot (for measures) are derived from article 'Pondera' (Smith's 'Dictionary of Greek and Roman Antiquities'). The Olympic cubic foot is to the $\frac{2}{3}$ Babylonian cubic foot, as 25 : 24, and equals 1,800 cubic inches; it would therefore be derived from Babylonia, and may be Akkadian (see p. 120).

The Etrurio-Roman cubic foot is $\frac{55}{54}$ of the Solonian, and equals 1,584 cubic inches; it appears to be derived from the Akkadians through the Etruscans. The name Etrurio-Roman is taken from Petrie's 'Inductive Metrology.'

The weight of the later Roman pound (5,235) is $\frac{16}{15}$ of the Solonian or Early Roman pound (4,909), agrees closely with the average of estimates of Arbuthnot, Greaves and Hussey, and is nearly identical with the modern Roman pound of 5,234 imperial grains (Kelly's 'Universal Cambist,' 1821).

The three English pounds, the Tower, Troy, and Avoirdupois appear to have been derived from different sources (p. 65 for pound Avoirdupois). The pound, as it should be by the test of 80 to a cubic foot of water, is as follows:

| | Ancient Grains. |
|---|---|
| The Babylonian commercial mina, log, or pound is 8,181 imperial grains ... | 7,200 |
| The English pound should be 5,454 imperial grains (the Tower pound) ... | 4,800 |
| The English pound actually is 5,760 imperial grains (the Troy pound) ... | 5,120 |

It will be shown that this pound of 5,454 imperial grains, when corrected for depreciation of grains, is the Tower pound of 5,400 imperial grains (see Chapter XI.), which was our Standard pound at the Mint until the reign of King Henry VIII. The Troy pound belongs to a different system.

# CHAPTER IX

## WEIGHTS

The use of grain for weights has been a custom among mankind from time immemorial. 'It would appear that even now the wild hillsmen of Annam weigh their gold-dust by grains of maize and rice. Dr. Edward Bernard ("De Mensuris et Ponderibus," Oxford, 1685) states that many of the ancients served themselves with ordinary grains of corn for the measures both of length and capacity' (Chaney, 'Our Weights and Measures,' p. 24).

One of our earliest statutes (51 Henry III., 1266, 'Assiza Panis et Cervisiæ') provided that the English silver penny, called the 'sterling,' should weigh '32 grains of wheat, well dried, gathered out of the middle of the ear.'

Weights appear to have originated independently of measures, but based on the same unit of measurement—the palm; at first the cubic palm, crammed with grains of barley, which were found to number on an average 4,000.

As time went on, weights were connected up with measures, and it was found that a cubic palm of water by weight was more easy to deal with than grains of barley for testing larger measures, and then was introduced the measurement by rain-water, the weight still remaining in grains. Thus a cubic inch of rain-water varied from about 220 grains of barley in Egyptian measure to $222\frac{2}{9}$ grains of barley in Babylonian measure.

Then it was found that the cubic palm of water did not fit in conveniently with the measures deduced from the talent, or $\frac{2}{3}$ (cubit cubed).

The cubic palm (25·51 cubic inches) of water weighs about 5,600 ancient grains, and 25·928 inches cube of water weigh 5,760 ancient grains, differing only by 160 grains from the true cubic palm (5,600).

This measure of 5,760 grains ancient was adopted as the standard measure of the ancients—*the conventional palm cube*—and for convenience it will be called the 'palm cube' in this chapter.

The reason why this particular measure was taken as a standard will be at once apparent when the calculations leading to it are shown.

There are 2,744 cubic palms to a double cubit cubed—*i.e.*, (14 palms)$^3$. The talent is $\left(\frac{2}{3}\text{ cubit}\right)$ cubed—*i.e.*, $\frac{2,744}{27} = 101·6$ cubic palms, or $\frac{70,000}{27}$ cubic inches. It was difficult to deal with 101·6 cubic palms, so the ancients took it as 100, and at the same time increased the cubic palm from 5,600 to 5,760 ancient grains, giving $100 \times 5,760 = 576,000$ to a talent, instead of $101·6 \times 5,600 = 568,960$, a discrepancy of about 1·2 per cent., so that instead of the cubic palm being 25·51 cubic inches it thus became 25·925 cubic inches $\left(\frac{70,000}{2,700}\right)$. The advantages, however, are obvious. There is no apparent difference between the actual palm cube and the conventional palm cube, and with the latter there are 100 to a talent and 27 talents to a double cubit cubed, and the number of grains in palms is $(3)^2 \times (4)^3 \times 10 = 5,760$. Thus the number of grains in a talent became 576,000, and in a double cubit cubed 15,552,000, instead of 15,400,000.

From this measure of 5,760 grains—the palm cube—both the Babylonian log and the Egyptian hon can be derived. It is the common measure of the ancients, the first weight, on which everything else is founded.

The log is $1\frac{1}{4}$ palm cube $= 5,760 + 1,440 = 7,200$ grains ancient.

The shekel is $\frac{1}{30}$ palm cube $= 192$ grains ancient.

The hon is $1\frac{1}{24}$ palm cube $= 5,760 + 240 = 6,000$ grains ancient.

The kat is $\frac{1}{48}$ palm cube $= 120$ grains ancient.

There is another small discrepancy, due to the different numbers used for division in the Babylonian and Egyptian systems.

6,000 ancient grains in a hon $\times$ 2,560 $=$ 15,360,000 grains in a double cubit cubed.

The conventional number is 15,400,000 grains in a double cubit cubed.

That is to say, the log is about 1 per cent. too large (as already shown), and the hon is about $\frac{1}{4}$ per cent. too small.

The reason of these discrepancies can be shown in another way. The log, as has been already shown in Chapter VI., is $\frac{1}{2,160}$ of double cubit cubed; the hon is, as shown in Chapter V., $\frac{1}{2,560}$ of double cubit cubed.

We have, therefore, the following calculations:

The kat through the Babylonian system ... ... ... $= \left\{\begin{matrix} \frac{2,744}{2,700} \times \frac{1}{48} \\ \frac{2,744}{2,160} \times \frac{1}{60} \end{matrix}\right\} = \frac{2,744}{12,960}.$

The kat through the Egyptian system ... ... ... $= \left\{ \frac{2,744}{2,560} \times \frac{1}{50} \right. = \frac{2,744}{12,800}.$

The difference is about 1·27 per cent., and is unavoidable.

There are 48 kats to the conventional palm cube. There would be about 47·2 kats of 120 grains to an actual palm cube (or 5,667 grains ancient), taking it through the Babylonian system; and 46·77 kats of 120 grains to an actual palm cube (or 5,612 grains ancient), taking it through the Egyptian system. In the first case the number of grains in bulk is 4,045, and in the latter 4,008·7 (originally 4,000). From this it is assumed that the Egyptian system is the earliest, giving 154,000,000 (originally $\frac{2{,}744 \times 4{,}000 \times 7}{5} = 15{,}366{,}600$) to a double cubit cubed, and 11,000,000 (originally $2{,}744 \times 4{,}000 = 10{,}976{,}000$) in bulk.

To recapitulate:

| | Grains Ancient. | | Grains Ancient. |
|---|---|---|---|
| 100th part of talent | = 5,760 | | 5,760 (or 48 kat). |
| Add $\frac{1}{4}$ | = 1,440 | Add ($\frac{1}{24}$) | 240 (or 2 kat). |
| 1 log | = 7,200 | | 6,000 = 1 hon = 50 kat. |

So that 1 log of 7,200 ancient grains = 32·406 cubic inches.

Also 6,000 ancient grains = 50 kat =

$$1 \text{ hon } \ldots \quad \ldots \quad \ldots = \begin{cases} 27\cdot & \text{cubic inches.} \\ 27\cdot3475 & \text{,,} \quad \text{,,} \end{cases}$$

These variations in the number of cubic inches are due to the discrepancy of 1·27 per cent. alluded to *ante*.

It may be well here to compare the number of shekels to each weight:

| | | | | |
|---|---|---|---|---|
| Shekel ... ... | 192 | ancient | grains | 1 shekel. |
| Palm cube ... ... | 5,760 | ,, | ,, | 30 shekels. |
| Log ... ... ... | 7,200 | ,, | ,, | $37\frac{1}{2}$ ,, |
| Hon ... ... ... | 6,000 | ,, | ,, | $31\frac{1}{4}$ ,, |
| Kat ... ... ... | 120 | ,, | ,, | $\frac{5}{8}$ ,, |
| Early Egyptian weight | 180 | ,, | ,, | $\frac{15}{16}$ ,, |

TABLE XIV.—COMPARING NUMBERS OF GRAINS ANCIENT IN A DOUBLE CUBIT CUBED TAKEN RESPECTIVELY THROUGH THE EGYPTIAN AND BABYLONIAN SYSTEMS.

| | Cubic Palms. | Cubic Inches. | No. in 70,000. | Ancient Grains. Egyptian System.* | Ancient Grains. Babylonian System. | Ancient Grains. Bulk. | Imperial Grains. |
|---|---|---|---|---|---|---|---|
| Inch cube ... | — | 1 | 70,000 | 220·0 | 222·2 | 158·28 | 252·458 |
| Palm cube ... | 1 | 25·51 | 2,744 | 5,612 | 5,667 | 4,000 | 6,476 |
| Conventional Palm cube { $\frac{1}{2700}$ double cubit cubed. } | 1 | 25·925 | 2,700 | 5,703 | 5,760 | — | 6,654 |
| 3-inch cube ... | — | 27·0 | 2,592 | 5,940 | 6,000 | 4,217 | 6,569 |
| Hon ... ... | 1·07 | 27·3475 | 2,560 | 6,015 | 6,075 | 4,296 | 6,903 |
| Log ... ... | 1·260 | 32·406 | 2,160 | 7,129 | 7,200 | { 5,104 5,094 } | 8,181 |
| Egyptian pint | 1·34 | 34·18 | 2,048 | 7,518 | — | 5,370 | 8 750 |
| Double cubit cubed ... | { 2,744 2,700 } | 70,000 | 1 | 15,400,000 | 15,552,000 | 11,000,000 | 17,622,000 |

## TROY WEIGHT.

On looking over our weights and measures it seems that the proportions may be very ancient, though the weight of the grains may have altered; and that the measures most likely to remain intact and unaltered should be those for precious stones and drugs, as it would dislocate all trade between nations in early times to make alterations in one, and it would affect the studies of the most learned branch of the community to alter the other. As an instance of the conservatism maintained in such matters, the following weights and liquid measures are set forth in the 'Pharmacopœia Collegii Regalis Medicorum,' Londinensis, 1851, as

* The number of grains under the Conventional system is given here. Under the Egyptian system there are only 219·5 ancient grains to a cubic inch, but it only alters the pound by from 15 to 20 grains.

those authorized to be used by Apothecaries, with the same names as were used in the times of Galen and Hippocrates; but the use of the grain in our system points to a still earlier period:

PONDERA.

Libra. Uncia. Drachma. Scrupulus. Granum.

MENSURÆ.

Congius. Octarius. Fluid Uncia. Fluid Drachma. Minimum.

Chaney ('Our Weights and Measures') says that apothecaries' weights, like apothecaries' measures, appear also to have had a Grecian and an Arabic origin.

The pound that is derived from our British yard or double cubit of 36 inches is the Tower pound of 5,400 grains imperial (5,454 grains corrected).

The Troy pound, the later Roman pound and the later kat (145 grains imperial) are respectively $\frac{16}{15}$ of the Tower pound, of the Solonian pound, and of the early kat (136 grains imperial).

Arbuthnot (1754) tells us that in his day there were 576 grains to the Cairo ounce, as in the Roman, French and Spanish ounce, but the latter less in weight, and that the number of 6,912 grains to the pound is the same in each.

In the Cairo ounce the grain in 1754 was almost identical with the Troy grain, and this was also the case in other parts of the East (as, for example, at Mocha, 'Kelly's U. Cambist,' 1824). The Vachia of Mocha was 160 carets (of 3 grains), and weighed almost exactly one ounce Troy.

An examination of 'Kelly's Cambist' will go far to show that there was one weight for grains all over Europe and the East, and that we all derive our inch and weight of grain from the $\frac{2}{3}$ (double cubit cubed).

In order to test the matter, our measures are put down alongside the kat and the shekel, and another Egyptian weight which may have been in use in early times:

| Ancient Grains. | | | Imperial Grains. | | |
|---|---|---|---|---|---|
| Kat. | Egyptian Weight. | Shekel. | Troy Weight. | Apothecaries' Weight. | Apothecaries' Fluid Measure. |
| Grains 120 =1 kat. | Grains 180 =? | Grains 192 =1 shekel. | Grains 24 =1 penny-weight. | Grains 20 =1 scruple. | — |
| — | — | — | Grains 120 =1 Anglo-Saxon shilling. | Grains 60 =1 dram. | Grains 60 =1 dram. |
| Grains 480 =4 kat. | Grains 480 =$2\frac{2}{3}$ | Grains 480 =$2\frac{1}{2}$ shekels. | Grains 480 =1 ounce. | Grains 480 =1 ounce. | Grains 480 =1 ounce. |
| Grains 5,760 =48 kat. | Grains 5,760 =32 | Grains 5,760 =30 shekels= 12 ounces. 15 ounces, 7,200. 20 ounces, 9,600. | Grains 5,760, or 12 ounces =1 pound. | Grains 5,760, or 12 ounces =1 pound. | 12 ounces, 5,760. 15 ounces, 7,200. 16 ounces, 7,680. 20 ounces, 9,600 = 1 pint or fluid pound (?) |

A carat, or caract, is an ancient weight of 4 grains; therefore there were 30 carats to 1 kat. A gerah was $\frac{1}{20}$ of a shekel, and does not seem to be a portion of the shekel of 192 grains. There was, however (as will be shown), possibly a commercial or non-sacred shekel of 160 grains, in which case the gerah would be 8 grains; thus measuring the gold shekel of 16 grains also.

### The Silver Shekel.

According to Gesenius, the shekel was a weight of gold or silver containing 20 beans (gerah—Exod. xxx. 13), which the Hebrews used, when weighed, for money (Gen. xxiii. 15, 16). Of this there are two kinds distinguished—the holy shekel (Exod. xxx. 13) and the royal shekel (2 Sam. xiv. 26).

The Hebrews certainly had no coinage before the Cap-

tivity, but there are allusions to a coin, or weight, which in the Revised Version is translated 'daric' (1 Chron. xxix. 7; Ez. ii. 60; viii. 27; Neh. i. 30).

In the time of the Maccabees silver coins were struck of the weight of a shekel (1 Macc. xv. 6), which contained 4 Attic drachmæ (*i.e.*, 1 stater), according to Josephus ('Ant.,' iii. 8, 2). Gesenius tells us that the Maccabean coins weigh 215 to 229 grains imperial, but the usual average given by various authorities is 218 grains.

The LXX. render the word 'shekel' by 'didrachma' instead of 'tetra-drachma,' which may be reconciled with the account of Josephus by supposing that the shekel was doubled after the time of the Maccabees from 192 to 384 grains ancient.

Conder's 'Handbook to the Bible,' p. 65 (Metrology) tells us that Maimonides states that the selah, or selang, containing 384 grains of barley, was (after the return from Babylon) substituted for the original shekel of 320 grains.

The later shekel of 384 grains would run 1,500 to a talent of 576,000 grains, instead of 3,000 (as with shekel of 192 grains).

This may clear up the difficulty mentioned by Madden ('Jewish Coinage,' p. 235) that the didrachma of the Old Testament (Septuagint) is a shekel and in the New Testament is half a shekel, the New Testament shekel being a tetradrachma.

If we may take 160 instead of 320 grains for the second shekel mentioned by Maimonides, we have a shekel running 3,600 to a talent, $\frac{576{,}000}{3{,}600} = 160$ grains ancient. This is a most probable number for a commercial shekel, as there are other records to show that the Babylonian talent was divided into $60 \times 60 = 3{,}600$ shekels. We have thus:

Shekel of sanctuary ... ... 3,000 to talent.
192 grains ancient.

Shekel royal or commercial... 3,600 to talent.
160 (? 320 after the Captivity) grains ancient.

Maccabean shekel ... ... 1,500 to talent.
384 grains ancient.

Madden ('Jewish Coinage,' p. 267) gives to the Babylonian shekel the weight of the Egyptian kat, 133·2 grains imperial, thus making $60 \times 60 \times 120 = 432{,}000$ grains to the talent, thus also arriving at the (log or) maneh of 7,200 grains ancient. We have then:

$$120 \times 60 \quad = 7{,}200$$
$$192 \times 37{\cdot}5 = 7{,}200$$
$$160 \times 60 \quad = 9{,}600, \text{ the higher mina.}$$

## Ancient Gold System.

It is not apparent how the idea arose among investigators that there was a large Hebrew gold talent heavier than that of silver. All seem to put the idea forward with diffidence, and the basis on which it stands seems to be very insecure.

Madden ('Jewish Coinage') supports his conclusions by references to Josephus, but he does not appear to think that they are convincing.

He says that Josephus states:

1. That the Hebrew talent of gold contained 100 minæ ('Ant.,' iii. 6, 7).

Josephus in this passage merely says that a gold candlestick weighed 100 litræ, or a talent; but this does not seem to imply that it was a golden talent.

2. That the Hebrew mina of gold was equal to 2½ litræ ('Ant.,' xiv. 7, 1).

In this, again, Josephus only states that a golden beam weighed 100 minæ, each of which weighed 2½ pounds.

3. That the golden shekel was a daric ('Ant.,' iii. 8, 10).

The passage is: 'Each head of a tribe brought a bowl and

a charger and a spoon of 10 *darics*, full of incense. Now, the charger and bowl were of silver, and together they weighed 200 shekels, but the bowl cost no more than 70 shekels.'

This might mean that 10 darics were equal in value to 200 shekels' weight of silver, or it might mean that the spoon was of gold, and equal in weight or value to 10 darics; but however it is taken it does not seem to imply that a shekel of gold was of greater weight than the shekel of silver.

This misapprehension about the size of the gold talent has caused the account of the wealth of the ancient world in gold to be much exaggerated, and has led to the assumption on the part of some that the amounts of gold quoted in some passages of the Old Testament are exaggerated.

For example, it is stated (1 Chron. xxii. 14) that King David allotted 1,000,000 talents of silver and 100,000 talents of gold for the building of the Temple. (*Note*.—Josephus divides both these amounts by 10—'Ant.,' 8, 14, 2.)

These amounts, as quoted in the Bible, are calculated by Mr. Napier (Smith's 'Dict. Bib.,' article, 'Metals') to amount to £939,929,687, and it is pointed out that all the gold in use in the world in 1851 only amounted to £820,000,000.

Mr. Napier is said to have arrived at his conclusions as follows:

| | |
|---|---|
| Silver talent ... 1,000,000 × 1,500 ounces | £ |
| × 52½d. ... ... ... ... ... | 328,125,000 |
| Gold talent 100,000 × 1,500 ounces × 73s. | 547,500,000 |
| | £875,625,000 |

It will be shown, however, that a talent of gold amounts to only 48,000 grains ancient, about 8 pounds Avoirdupois = to, say, 100 ounces at 73s. = £365. Therefore 100,000 gold talents would value £36,000,000 instead of £547,500,000, a reduction of from 15 to 1.

A reference to the Old Testament will probably afford a complete solution of the difficulty about the weight of the talent of gold. There are parallel passages which have hitherto been considered ambiguous, and which by some have been supposed to be inconsistent with each other:

David paid for the threshing-floor and oxen of Ornan 50 shekels (2 Sam. xxiv. 24).

David purchased the threshing-floor of Ornan 'for 600 shekels of gold *by weight*' (1 Chron. xxi. 25).

Now, silver was the medium of exchange, but gold was used for weighing, and there were shekels of gold as well as shekels of silver. The problem is to discover their relation one to the other.

We have 50 shekels of silver *by weight* = 600 shekels of gold *by weight*.

∴ 1 shekel of silver *by weight* = 12 shekels of gold *by weight*.

*I.e.*, that at the time of King David the relation of gold to silver was as 12 : 1 in value weight for weight.

Now we know from p. 81 that there were 3,000 silver shekels to the silver talent, and that by taking 576,000 grains of barley to the talent we have a shekel of silver of 192 grains of barley.

∴ 192 grains of barley by weight = 12 shekels of gold by weight.

∴ 1 shekel of gold weighed 16 grains of barley (ancient).

Therefore, if we may assume 3,000 shekels of gold to a golden talent, we have its weight 48,000 grains of barley instead of about 1,200,000, as usually supposed. Of the relative *values* of gold and silver we cannot be quite certain, but we can make inferences.

Of one point we may be certain, that the stater or daric of the earliest coinage, of the weight of a kat or 120 grains ancient, was 7½ shekels of gold.

Now, as the relation was 12 silver to 1 of gold,

A golden daric of $7\frac{1}{2}$ gold shekels, and weighing 120 grains of barley, was equal in value to silver coins weighing 1,440 grains of barley.

But $7\frac{1}{2}$ shekels of silver weigh 1,440 grains of barley.

$\therefore$ $7\frac{1}{2}$ shekels of gold $= 7\frac{1}{2}$ shekels of silver (in value).

$\therefore$ 1 shekel of gold $=$ 1 shekel of silver (in value).

$\therefore$ 3,000 shekels of gold, weighing 48,000 grains of barley (or grains ancient), making 400 darics, are equal in value to a talent of silver, or 3,000 shekels of silver of 192 grains ancient each.

That is to say, 3,000 shekels of gold at 16 grains each are equivalent in value to 3,000 shekels of silver at 192 grains.

This seems to be a complete solution of the matter, and it reduces the estimated amount of gold in early times to reasonable proportions.

It is to be noted that a shekel of gold being 16 grains, and silver being in bulk to gold as 12 : 16—

A shekel of *gold in bulk is* 12 *grains weight of silver.*

To ascertain the bulk and weight of a talent of silver and gold roughly: A talent of silver weighs 655,000 grains imperial $= \frac{655{,}000}{252\cdot4}$ cubic inches of water. The specific gravity of beaten silver and distilled water were respectively 18·5 and 1·75; say $10\frac{1}{2} : 1$.

$$\text{Silver talent } \frac{655{,}000}{252\cdot4 \times 10\cdot5} = \frac{131{,}000}{252\cdot4 \times 2\cdot1} = \frac{131{,}000}{529\cdot1}$$

$$= \left\{ \begin{matrix} \text{250 cubic inches of silver,} \\ \text{a little more than a cube of 6 inches} \end{matrix} \right\} \begin{matrix} \text{in weight 93·57} \\ \text{pounds Avoirdupois.} \end{matrix}$$

$$\text{Gold talent } \frac{48{,}000}{220 \times 16} = 13\cdot6 \text{ cubic inches of solid gold}$$

(a cube of less than $2\frac{1}{2}$ inches). Weight 7·79 pounds Avoirdupois nearly.

### The Miṅa.

Maṅeh, mina (Greek μνᾶ), is supposed to have been the weight of 100 shekels (1 Kings x. 17; 2 Chron. ix. 16): '300 shields of beaten gold, 3 minas of gold went to 1 shield'; '300 shields of gold, 300 shekels of gold went to 1 shield.'

Now, 600 shekels of gold (p. 104) weighed 50 shekels of silver, or 9,600 grains ancient; the mina of gold would therefore weigh 1,600 grains of barley.

As there are 16 grains of gold to a shekel, the mina of gold would be 100 shekels of gold, and 30 minas of gold, or 3,000 shekels, would be a talent of gold.

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From the knowledge we have now of the silver shekel and mina, an obscure passage in Ezekiel (xlv. 12) can be cleared up: '20 shekels, 25 shekels, 15 shekels shall be your maneh.'

He spoke of the later or double shekel of silver of 384 grains imperial. According to previous usage they would have run:

$18\frac{3}{4}$, 25, 15 shekels to the mina.

*I.e.*, $18\frac{3}{4} \times 384 = 7{,}200$ grains ancient. The log, 15 ounces ($15 \times 480$).

$25 \times 384 = 9{,}600$ grains ancient. The heavy mina, 20 ounces ($20 \times 480$).

$15 \times 384 = 5{,}760$ grains ancient. The palm cube, 12 ounces ($12 \times 480$).

The change that Ezekiel made is exactly what has taken place in later times. He changes the 15 ounces of the log to 16 ounces by adding 480 grains to the log (7,200 grains), thus making it 7,680. Thus the proportions became 16, 20, 12 ounces (or 20, 25, 15 shekels), as at present:

The principal weights in use at the present day are 12, 16 and 20 ounces (5,760, 7,680, 9,600 grains, p. 99).

From this we may suppose that the ounce of 480 grains was a measure both with the Babylonians and also with the Egyptians.

### The Egyptian Kat.

The kat seems to be the most early weight of which we have record, weighs 120 ancient grains, and should average 136 imperial grains. In later days it seems to have risen to 145 grains imperial or $\frac{16}{15}$ (135).

The silver shekel weighs 192 ancient grains, and should average 217 to 219 imperial grains.

An Egyptian weight weighs 180 ancient grains, and should average 196 to 197 imperial grains.

### Weights (principally from Madden's 'Jewish Coinage').

| Imperial Grains. | Ancient Grains. | | |
|---|---|---|---|
| 140 | 120 | Stone weight, 698 grains imperial, 5 kats | 'Soc. Bib. Arch.,' vol. xiv. |
| 140½ | 120 | Stone weight, 1,404 grains imperial, 10 kats | |
| 135 | 120 | Homeric, Roman, Sicilian ox-weights (Chaney, 'Our Weights and Measures'). | |
| 133·3 | 120 | Nineveh weights, 2 minas, 700 B.C., 15,987 grains imperial. | |
| 140 | 120 | Weight from Jerusalem, 42,000 grammes ('Du Bimétallisme chez les Hébreux'). | |
| 128·6 | 120 | Daric, gold Persian (British Museum). | |
| 132·5 | 120 | Stater, gold Attic (after 335 B.C.). | |
| 129 | 120 | „ gold Lampsacus (434 B.C.) (British Museum). | |

| Imperial Grains. | Ancient Grains. | |
|---|---|---|
| 133·0 | 120 | Stater, gold Macedonian (330 B.C.). |
| 130 | 120 | Aureus, gold Roman (40 to the Roman pound). |
| 126·7 | 120 | $\frac{3}{2}$ Sigli, Persian, 600 B.C. |
| 133·3 | 120 | $\frac{3}{2}$ Duck weight. |
| 140 | 120 | Didrachma, copper, Egyptian, Ptolemy. |
| 140 | 120 | Didrachma, copper, Maccabean. |

### List of Weights of Shekels of Silver.

| Grains Imperial. | Grains Ancient. | |
|---|---|---|
| 218 | 192 | Didrachma, silver Eginetan (see note, p. 82). |
| 218 | 192 | Shekel, silver Maccabean. |
| 220 | 192 | „ „ Tyre (500 B.C.). |

It is to be noted that whilst weights depreciate very slowly, coins constantly vary in weight, and are of little value by themselves to give accurate absolute values of weights unless they have been struck for use as weights, as in the case of the shekels.

### Copper.

According to the proportion of silver to copper (money) : : 60 : 1, the copper talent should be 60 silver talents, or 6,000 palms cube. The copper shekel would therefore (at 3,000 to the talent) be 2 palms cube. We have thus the following proportions :

| | | | |
|---|---|---|---|
| Shekel—Gold | ... | 16 | grains ancient. |
| „ Silver | ... | 192 | „ „ |
| „ Copper | ... | 11,520 | „ „ |

To put it another way:

> An ounce (480 grains) of gold = a pound (5,760 grains) of silver = 60 pounds of copper.

As to the divisions of copper into money, the fluctuations from age to age were so great that it is difficult to obtain any satisfactory results.

Egyptian copper coins of the time of the Ptolemies weighed a kat each, or a subdivision of a kat, and there are 96 kat in 60 shekels of silver.

It does not seem to be known how the shekel was subdivided in copper, but if we may consider 24 as a divisor, we shall have 4 kat, or coins equal in weight to our modern penny, to a twenty-fourth of a silver shekel.

Madden gives examples of Jewish coppers of weight 132 grains imperial (120 ancient), marked one quarter.

In the pamphlet 'Du Bimétallisme chez les Hébreux,' in which the values of Hebrew and Roman weights and measures of capacity and the weights of the gold and silver minæ are given by the Vicomte François de Salignac Fénelon, the estimates are founded on the supposition that the Maccabean shekel weighed exactly 14 grammes, and was the three-thousandth part of a talent of 42,000 grammes. In this way he arrives at measures which differ about 1 per cent. from those given in this work derived from the cubit of the Great Pyramid. Had he taken the full weight of the shekel, the measures and weights would have closely agreed. 3,000 × 192 gives 576,000 grains ancient; 42,000 grammes gives 648,159 grains imperial, about 578,000 grains ancient; but of course 42,000 is a round number, and would not exactly coincide with the ancient measure. He founds his proposals also on the weight (about 42,000 grammes) of a large stone weight found at Jerusalem.

## CHAPTER X

### DERIVATIVES OF THE ANCIENT CUBIT OF $20{\cdot}61\dot{0}\dot{9}$ INCHES

If with the content of the double cubit cubed (70,000 cubic inches) we take consecutively 18, 12, 11, 9, 8, 7, 6, and 5 tenths, we have the double cubits cubed, or contents, of a new set of cubits cubed; and by extracting the cube roots we arrive at the lengths of these cubits as follows: $\frac{18}{10}$ gives 25·065; $\frac{12}{10}$, 21·89; $\frac{11}{10}$, 21·26; $\frac{9}{10}$, 19·89; $\frac{7}{10}$, 18·29; $\frac{6}{10}$, 17·38; $\frac{5}{10}$, 16·34 inches. If we take $\frac{6}{7}$ we get 19·57 inches, and $\frac{64}{70}$ gives 20 inches. Again, $\frac{4}{6}$ and $\frac{5}{6}$ give 18 and 19·39 inches.

When these are compared with the cubits deduced by Mr. F. Petrie from existing remains all over the world ('Inductive Metrology'), it will be found that the whole of the cubits he deduces are accounted for except those mentioned under columns 'Digit,' 'Copan,'* and 'Various.' A table is here attached comparing the results obtained with those of Mr. Petrie.

So far as we may judge of their priority, that of $\frac{6}{7}$ is likely to be ancient. It is the cubit of 19·57 inches, known at present as the cubit of Gudea, found at Telloh, near Babylon. It stands by itself, but is connected up, as will be seen, with the ancient cubit. Next come a batch

* The architectural remains of Copan, in Central America, give a unit of 6·817 inches, which I take to represent the $\frac{1}{3}$ Babylonian cubit of 6·87 inches.

of 4 cubits—25·065, 21·89, 21·26, and 19·89—called by Mr. Petrie respectively the Royal Persian and Hebrew and Chaldean, the Phœnician (foot), the Assyrian cubit, and the half Assyrian great U. This disposes of all the ancient cubits, and it is worth remarking that the Phœnician cubit or foot is found in the rude-stone monuments of Great Britain and France.

TABLE XV.

| | Content. Cubic Inches. | Derived Cubit. | Derived Foot. | Petrie's Cubits. Maximum. | Petrie's Cubits. Minimum. | |
|---|---|---|---|---|---|---|
| 7 | 70,000 | $20{\cdot}61\dot{0}\dot{9}$ | 13·7406 | — | — | Ancient royal cubit of Egypt and Babylon from the Great Pyramid. |
| 6 | 60,000 | 19·57 | 13·05 | — | — | Cubit of scale of Gudea from Hommel. |
| 10 | 70,000 | $20{\cdot}61\dot{0}\dot{9}$ | $13{\cdot}74\dot{0}\dot{6}$ | 20·76 | 20·6 | Ancient royal cubit from the Great Pyramid. |
| | | | 12·36 | 12·47 | 12·4 | Babylonian foot ⅗ (20·6109). |
| 18 | 126,000 | 25·065 | — | 25·38 | 25·1 | Royal Persian, Hebrew, and Chaldean. |
| | | — | 16·710 | 16·89 | 16·66 | Aretni [royal foot ⅔ (25·065)]. |
| 12 | 84,000 | 21·89 | — | 22·5 | 21·48 | Rude stone monuments of Great Britain and France; Phœnician foot (?). |
| 11 | 77,000 | 21·26 | — | 21·40 | 21·30 | Assyrian cubit. |
| 9 | 63,000 | 19·89 | 13·22 | 20·24 | 19·90 | Half Assyrian great U. |
| 7 | 49,000 | 18·29 | 12·19 | 12·23 | 12·11 | Olympic foot. |
| 6 | 42,000 | 17·38 | 11·58 | 11·74 | 11·51 | Ancient Greek and Roman foot. |
| 5 | 35,000 | 16·34 | 10·89 | 10·92 | 10·80 | Plinian foot. |
| 6 | 70,000 | — | — | — | — | Ancient royal cubit, as above. |
| 5 | 58,333 | 19·39 | — | 19·30 | 18·92 | Double Pythic foot. |
| 4 | 46,665 | 18·00 | 12·0 | 17·90 | 17·82 | Hasta (British foot). |
| 35 | 70,000 | — | — | — | — | Ancient royal cubit, as above. |
| 32 | 64,000 | 20·0<br>16·0 | $13{\cdot}\dot{3}$ | 13·45 | 13·16 | Drusian foot, synagogues of Syria. |

We then have three cubits of 18·29, 17·38, and 16·34 inches, from which the Olympic, ancient Roman, and Plinian foot are derived.

Distinct from these are those of $\frac{2}{3}$ and $\frac{5}{8}$, the 19·39 (double Pythic) and 18 inches (Hasta?), probably ancient Grecian cubits (and modern English). Distinct again are two cubits 20 and 18·88 inches, based apparently on the Drusian foot. The 16-inch cubit of the 'Handbook to the Bible' is evidently the same as the 20-inch cubit, as they both measure the same synagogues, being in proportion 5 : 4. It is possible that this is the half Assyrian great U, 19·89, but Mr. Petrie considers the unit of 18·8 distinct from that of 18·2. The contents of these latter double cubits cubed are as 64 : 68. Possibly the 16 and 20 inch cubits are variations of the British 18 inch cubit or 12 inch foot (p. 92).

THE CUBIT OF GUDEA, B.C. 2500 TO 2800 (? DOUBTFUL).

On the sitting statue of Gudea found at Telloh (Larsa), in Southern Babylonia, is a plan of a town, and alongside it a scale known as the scale of Gudea (De Sarzec's 'Découvertes'). There is no certainty what the scale means, but the strong presumption is that it indicates the linear measures of that period for building purposes, and as it is the only vestige of an early linear measure yet found out of Egypt, it is naturally a scale of considerable importance in investigations of the lengths of early cubits. It may have been used only locally or throughout the land; it may have been used for temples or for all building purposes; but, whatever purpose it was designed for, it is evidently a linear measure of some kind.

If the plan of the town on the Lap of Gudea be compared with the plan of the existing buildings at Telloh (Maspero's 'Dawn of Civilization,' p. 711), it will be seen that all the gates coincide in position.

I propose to compare the measures given on this scale with the ancient cubit of 20⅝ inches recorded in the Great Pyramid of Gizeh centuries before the supposed date of the scale of Gudea, and to do so I will first make use of measurements given by Professor Hommel in article 'Babylonia' (Hasting's 'Dictionary of Bible,' 1898). From the measurements of the scale of Gudea M. Hommel deduces the double cubit of that period as lying between 990 and 996 millimetres, possibly coincident with the length of the seconds pendulum in the latitude of Babylon (about 30° N.)—viz., 992·35 millimetres (39·07 inches)—and he arrives from this at the standards of linear, square, and cubic measures and weights of the period. It will be shown how far these results seem to agree with measures derived from the ancient cubit of Babylon and Egypt.

I take this cubit (as deduced by M. Hommel from the scale of Gudea) to have been derived from the content of the ancient double cubit (41¼) cubed (70,000 cubic inches) by taking *six* out of seven parts, and extracting the cube root of that quantity (60,000 cubic inches) = 39·1487 linear inches. This I take to be the double cubit of Gudea; it equals about 994 millimetres, and thus lies between the 990 and 996 millimetres given by M. Hommel. He states that this double cubit was divided into ten parts, and that the tenth part is the side of a cube containing a *ka*, when filled with water (weighing 990 grammes, or 15,277 grains imperial). A cube on this tenth part (3·9148 inches) will contain exactly 60 cubic inches of water, and 360 of these (the *gur*) amount to 21,600 cubic inches. Assuming the *ka* to be the double mina of these people, the mina would equal 7,579 grains imperial, and therefore (at 60 shekels to the mina) the double shekel would equal 252·5 grains imperial (222⅞ ancient grains of Babylon, or 220 ancient grains of Egypt), or the weight of a cubic inch of rain-water.

In article 'Money' (Hasting's 'Dictionary of Bible') a weight is spoken of as being deduced by Dr. Lehmann from existing Babylonian weights weighing 7,580 grains imperial (agreeing with my deduction), or half the weight of a ka of water. On this, as a mina, is based a system, 60 minas to the talent and 60 shekels to the mina and 180 shé to the shekel, giving 3,600 shekels to the talent. (For shé see p. 119.)

The matter requires some explanation.

Professor Kennedy (the writer of the article 'Money') has taken a value of 126* grains imperial to the shekel, *i.e.*, $\frac{1}{60}$ (7,580) grains imperial, which he terms the common standard, and all his calculations are based on this standard.

That is to say, 60 shekels to the mina, and again 60 minas to the talent, thus giving $126 \times 3{,}600 = 453{,}600$ grains imperial to the talent. This is the common scale, having a series double the weight also for commercial transactions.

For weighing precious metals another scale was used. On the gold scale the shekel of 126 grains imperial was retained, but there was a mina of 50 shekels (6,300 grains) substituted, and 60 minas to the talent, thus giving 3,000 shekels (of 126 grains) to a talent of 378,000 grains imperial.

We now come to the connecting up with the ancient system. For the silver scale the whole system was altered in the proportion of 4 : 3. In other words, a shekel of 168 grains was introduced, 50 to the mina, thus equalling 8,400 grains. A talent consisted of 60 minas = 504,000 grains imperial.

It is then pointed out that the Phœnician shekel of 224 (?) grains (giving a mina of 13,440 grains imperial) bore to the heavy Babylonian shekel the ratio of 2 : 3. The results of this system of Gudea are tabulated below; but I concur only in the first (or Gudea trade) talent; I think that the others are about 3 per cent. too high in weight.

* Omitting fractions.

| | Grains Imperial. | | Grains Imperial. | | Grains Imperial. |
|---|---|---|---|---|---|
| Gudea trade talent | 454,800 = 60 minas. | | 7,580 = 60 shekels | | 126 |
| | 909,600 = | ,, | 15,160 = 60 | ,, | 252 |
| Gold scale | 378,900 = | ,, | 6,315 = 50 | ,, | 126 |
| Silver scale | 504,000 = | ,, | 8,400 = 50 | ,, | 168 |
| Phœnician | 336,000 = | ,, | 5,600 = 50 | ,, | 112 |

It is pointed out by the writer of the article 'Money' that $\frac{10}{9} \times \frac{1}{60}$ of light mina = Egyptian kat at 140 grains imperial.

The gold system is based on the view that the value of gold to silver by weight was as 40 : 3, or $13\frac{1}{3} : 1$; and the weight of the silver talent was raised till it stood in the proportion to gold of 4 : 3. So that a given weight of gold was *always equal to ten times the same weight of silver.* Thus there are two standards of weight, and the system is not so simple as the Hebrew system I propose, of having a different weight to the gold and silver shekels.

It is difficult to ascertain whether the various weights found in Babylonia support this proposed system of Gudea, for they vary so greatly one from another—as much as 8 per cent.: so that the mina weighs from 7,451 to 7,992 grains imperial. I think it probable that the heavier mina belongs to the Babylonian sexagesimal system, while the lighter mina belongs to the Gudean system. Most of these minas are as late as the seventh century B.C.

There is an ancient hæmatite weight from Nippur of ten shekels, weighing 1,320 grains imperial, which gives a shekel of 117·3 grains ancient, possibly intended for 120 grains, which does not seem to conform to either the Babylonian or proposed Gudean system, but rather to the Egyptian kat.

8—2

There are two weights with Semitic characters, weighing respectively 39·2 and 154 grains imperial, found in Palestine, of a date at least 500 B.C. If these are taken as having originally been respectively, 40·9 and 163 grains imperial, they would give a shekel of 144 ancient grains, and would thus agree nearly with the shekel of silver scale given in article 'Money' (see last page).

It seems to me that an element of uncertainty has been introduced in article 'Money' by taking the Phœnician shekel at 224 grains imperial, whereas the average seems to be from 218 to 220 grains imperial (p. 81), the kat being taken at 140 grains imperial instead of 136 grains imperial. The result is that the weights are much heavier than the early weights taken by me.

I think that the weight of 7,580 grains imperial gives a double shekel of a cubic inch, a silver mina of 30 cubic inches, and a talent of 1,800 cubic inches (the Olympic cubic foot). The talent, silver mina, and shekel agreeing exactly with those given in article 'Money' above referred to. (See Table XIII. and p. 93.)

My proposal is that this is an Akkadian or Sumerian system, and not Semitic, and that it has found its way into Asia Minor, Greece, and Etruria in early days.

Two of the weights, of 110 grains ancient each, went to the cubic inch, and sixty of them amounted to 6,600 grains ancient. By taking two-thirds, as with the $\frac{2}{3}$ Babylonian weights (Table XIII.), we arrive at a weight of 4,400 grains ancient, equal to 4,999 grains imperial.

$$(4{,}400 \times \frac{9}{5} + 1 \text{ per cent.} = 4{,}999).$$

This is nearly exactly the Roman pound (Table XIII., p. 92), which, I think, was received by the Romans from the Etruscans.

This Sumerian weight of 4,400 grains is a multiple of 11, being founded on 220 grains to the cubic inch.

This brings me to the introduction of a subject which I will only touch on, viz., the use of 11 in the weights and measures and calculations of Europe in medieval times.

The use of the number 11 in our Anglo-Saxon and Danish accounts in the eighth to the tenth centuries may be derived from the use of a weight of 110 grains (220 grains to a cubic inch), and also to the use of a length for square measure of 220 double cubits (p. 66).

On reference to the 'Tribal Hidage' (*Trans. R. Hist. Soc.*, 1900), it will be seen that the whole division of England into hidage depends on an elaborate system of calculation by eleven. For example:

| | Hides. | | |
|---|---|---|---|
| Bernicia - - | 30,000 | 44,000 | 66,000 |
| Dera - - - | 14,000 | | |
| South Humbria - | 10,000 | 22,000 | |
| Mercia - - - | 12,000 | | |
| East Anglia - - | 30,000 | 44,000 | 66,000 |
| Essex - - - | 7,000 | | |
| Hwicca - - - | 7,000 | | |
| Middle Anglia - | 12,000 | 22,000 | |
| Wessex - - - | 10,000 | | |
| Kent - - - | 15,000 | | 22,000 |
| Sussex - - - | 7,000 | | |
| | | | 154,000 |

The whole amounting, curiously enough, to one-hundredth part of the number of grains ancient in a double cubit cubed (p. 67, Table VII.).

I now give an independent calculation of the length of Gudea's cubit, derived from examination of De Sarzec's plates.

### Derivation of the Cubit of Gudea, the Akkadian.

This scale is derived from the double cubit of the Great Pyramid by eliminating the number 7—*i.e.*, by taking $\frac{6}{7}$ of the cubic contents, and $\frac{20}{21}$ of the linear measure. [*Note.*—Pyramid double cubit $= 14$ palms of $\frac{53}{18}$ inches each.]

(*a*) The double cubit cubed of Gudea

$$= \frac{6}{7} \text{ (double cubit cubed of Great Pyramid (70,000) cubic inches,}$$

$$= 60{,}000 \text{ cubic inches.}$$

(*b*) The double cubit of Gudea

$$= \frac{20}{21} \text{ (double cubit of Great Pyramid, } 41\tfrac{2}{9} \text{ inches),}$$

$$= \frac{1060}{27} = 39{\cdot}256 \text{ inches.}$$

This hinges upon the fact that $\left(\frac{20}{21}\right)^3$ nearly equals $\frac{6}{7}$; difference less than $\frac{1}{100}$.

| | | |
|---|---|---|
| The double cubit as thus derived is in (*a*) | $\sqrt[3]{60{,}000} = 39{\cdot}148$ inches. | |
| The double cubit as thus derived is in (*b*) | $= 39{\cdot}256$ | ,, |
| | Difference | $-0{\cdot}108$ |

The cubit of Gudea ($39{\cdot}256$) is to the Pyramid cubit ($41\frac{2}{9}$) as 20 : 21—*i.e.*, 21 Gudean cubits $= 20$ Pyramid cubits.

If the Gudean double cubit is divided into 60 equal parts, nine of these parts will equal 2 palms of $\frac{53}{18}$ inches.

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The scale of Gudea, as shown on the De Sarzec's plates 14 and 15, is divided into 16 parts, and something over. Total

length of scale written on plan by De Sarzec 27M = 10·6299 inches.

As measured on paper = 10·666 inches.

Difference 0·036 „

Total length of 16 divisions measured on the paper plan = 10·510 inches.

Deduct 0·036 „

True length of 16 divisions = 10·473 „

∴ length of 1 division = 0·654 „

∴ length of 63 divisions = 41·2398 inches (Pyramid cubit $41\frac{2}{9}$).

(Gudean cubit) 60 divisions = 39·276 inches (by calculation 39·256).

$\frac{9}{2}$ divisions = 2·945 inches (Pyramid palm 2·945).

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It is to be noted that 60 Babylonian pounds, of 8,181 grains imperial each, give a talent of 490,860 grains imperial, corresponding to the talent of 480,000 grains imperial (founded on a mina of 8,000 grains imperial) given in article 'Weights,' Hastings, 'Dictionary of the Bible,' and in 'Hebrew Weights,' by Colonel Conder, in 'P.E.F.Q.S.,' April, 1902.

The talent of 490,860 grains imperial gives the later Babylonian foot of 12·48 inches ($\frac{3}{5}$ Babylonian double cubit). This foot is found in architectural remains in Babylonia, Asia Minor, and Greece. (See 'Inductive Metrology.')

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If the shé (see p. 14) is taken as a grain of wheat equal to $\frac{3}{4}$ grain imperial, then 180 shé equal 135 grains imperial, or 120 grains ancient (the kat). It would appear, therefore, that the shé belongs to a late Babylonian sexagesimal, and not to the Akkadian system. The grain Troy (250 to the cubic inch) may have come into use in parts of Assyria and Asia Minor as early as 700 B.C.

## THE LATER HEBREW CUBIT OF HEROD'S TEMPLE.

The account given in the Talmud of the measurements of the Temple is all of a very late date, and it is evident that for the most part it rests upon conjecture and is very defective.

That the early cubit used by the Hebrews was $20\frac{11}{18}$ there seems to be no doubt, but whether in later times (on the rebuilding of the Temple by Herod) a smaller cubit was used we have little to guide us except the very uncertain statements in the Talmud.

The mountain of the house was divided into two parts, absolutely distinct, and it is possible that one cubit may have been used for the holy ground and another for the Court of the Gentiles and the outer cloisters. In any case, it is worth finding out what grist there is in so much chaff.

Just let us try and ascertain what is a handbreadth according to the Talmudists. Is it the original handbreadth of four fingers, as already given, or is it some artificial measure?

Dr. Lightfoot, in the 'Prospect of the Temple,' tells us (quoting from 'The Talmud') that the handbreadth is '*the four fingers as they are laid close together*, which make but 3 inches.' This, then, evidently is the 2·94 inches handbreadth of the ancient cubit. He then tells us that the Temple was measured by a cubit of 6 handbreadths and the vessels measured by the cubit of 5 handbreadths. He calls the 6 palm cubit the 'cubit of Moses,' and then mentions that there were two others: one $6\frac{1}{8}$ palms and another $6\frac{1}{4}$ palms. He further proceeds to say that the cubit of a handbreadth of Ezekiel is a cubit of $(5+1)$ palms, instead of $(6+1)$ palms, as taken in this work. This is the whole question. Was the later Temple of the Jews measured by a cubit of 6 or 7 palms, a cubit of $17\frac{2}{3}$ or $20\frac{11}{18}$ inches?

# CHAPTER XI

## THE GRAIN, THE MARK, AND THE POUND

### Grain as a Weight

Wheat does not appear to have been in use in early times in Babylonia or Egypt. Barley was used for food, and was used for weights also before wheat.

Subsequently, when wheat came into use, it was also made use of for weights, and the relative weights of an average grain of barley and of wheat in early times is required.

In our Earliest English statutes we find 24 grains to a pennyweight or sterling, and 32 grains of wheat to a sterling. The assumption, therefore, is that in those days four grains of wheat equalled three grains of barley.

So far as I can gather, this is about the average at the present day, although there are now so many varieties of wheat and barley that it is difficult to strike an average.

The following has been sent me as an independent calculation. '"A Bushel of Corn" gives the mean weight of the grains in seven samples of White and Red Spring wheat at ·55 Imperial grains, and states that the average weight of common barley is ·72 Imperial grains, *i.e.*, 72 wheat corns = 55 barley corns, or approximately 4 wheat corns to 3 barley corns.

'As an example, however, of the difference in the weights (depending partly on the dressing), it may be mentioned that the number of grains per pound in certain descriptions of wheat and barley given in "A Bushel of Corn" vary in the

case of wheat from 20,231 to 6,434 grains, and in the case of barley from 13,699 (bere barley) to 6,481 grains per lb.

'Red Rivet wheat, which was anciently cultivated in England, is stated to average ·992 Troy* grains per corn, while Italian barley averages ·89; chevalier, ·68; and bere, ·58 grains.'

The Rev. J. Edward Hanauer has kindly sent me (March, 1902) the following notes from Jerusalem on the weights of grain. '24 grains of barley are exactly equal to 29 grains of wheat. 4,000 grains of barley weigh 4,840 grains Troy, whilst 4,000 grains of wheat weigh 3,857 grains Troy. A box, three inches cube, of barley, contains (by weight) 4,903 grains Troy, and of wheat 5,539 grains Troy.

'Special attention must be drawn to the fact that in Syria weights of wheat and barley vary according to the locality they come from, and their ages. There is a very appreciable difference in weight of corn grown in the Hauran, the Belka, Tubas, and the Gaza districts. Wheat taken from the threshing-floor is different in weight from any in hand belonging to the crop of the preceding year. The crops of last year were a complete failure in Palestine, and consequently the corn, of which I have given you the weight, was old, possibly even two years old. It was procured in the market, and it is impossible to tell from what part of the country it comes.'

It seems certain from this that the wheat and barley weighed at Jerusalem was much lighter than it would have been when taken from the ear, having lost possibly as much as 20 per cent. of weight.

So far we may gather that a grain of barley was probably (when taken from the ear) at least as heavy as, or heavier than, the weight of a grain Troy; and that the proportions barley and wheat by weight may have been as 4 : 3 : but there is nothing very decisive on the subject. Professor Ridgeway

* Troy grains and imperial grains are the same weight.

('Origin of Currency and Weight Standards,' p. 180) gives much evidence of this proportion by weight of barley to wheat (4 : 3), and he gives the weight of a grain of Scotch wheat (from trials made by himself) as about ·71 grains Troy.

The next question is the relative weights of barley-grain and water to a cubic inch. I made several trials of ordinary English barley in 1901, of very inferior quality.

I found that 8,560 grains went to one pound Avoidupois, 20·52 cubic inches of this grain weighed about 8·45 ounces, and contained about 4520 grains.

∴ 27·7 inches (a cubic lb. Av.) contains 6,099 grains ot barley.

| | Water. | | Barley in bulk. | |
|---|---|---|---|---|
| Therefore the barley runs | 8,560 | to | 6,099 | grains imperial. |

(8,560 as against 7,000 grains imperial.)

The proportion is 8,560 : 6,099 :: 7 : 5 nearly.

| | | | |
|---|---|---|---|
| 27·7 cubic | inches | weigh | 8,560 of this common English barley. |
| ,, | ,, | ,, | 7,000 of grains imperial. |
| ,, | ,, | ,, | 6,015 of ancient grains. |

This trial was to ascertain whether the conventional proportion 7 : 5 (pp. 11, 64, 71) was likely to be approximately correct, and not to establish the weight of a grain of barley.

The next point is to show precisely the number of grains of barley (ancient) to a cubic inch, according to Babylonian, Egyptian and conventional measure (p. 78).

Babylonian measures; 15,552,000 grains to
70,000 cubic inches $= 222\frac{2}{9}$ grains ancient.

Conventional measures; 15,400,000 grains to
70,000 cubic inches = 220 grains ancient.

Egyptian measures; 15,360,000 grains to
70,000 cubic inches = 219·42 grains ancient.

Giving a difference of 1·27 per cent. between Egyptian and Babylonian measures (p. 96).

I have at p. 116 suggested that the Akkadians or Sumerians worked with 220 grains to the cubic inch; but the Babylonians were forced to work with a very inconvenient number of grains ($222\frac{2}{9}$), or else to change the number of grains to a cubic inch.

Now what is the most convenient number for a change, ranging about $222\frac{2}{9}$? Obviously a proportion which contains 9, so as to get rid of the fraction. Now $222\frac{2}{9}$ is actually $\frac{2000}{9}$, and therefore the nearest approach to $222\frac{2}{9}$ is 250.

$$\frac{2000}{9} : 250 :: 8 : 9.$$

There is nothing to show when this change took place, perhaps as early as 700 B.C.; the number of grains to a talent was thus reduced as $3 : 2 :: 576{,}000 : 384{,}000$, and then was increased $8 : 9 :: 384{,}000 : 432{,}000$, which is equivalent to a change of $4 : 3 :: 576{,}000 : 432{,}000$.

Thus there were 432,000 reduced grains to a talent, and 250 grains to a cubic inch, but there has been a loss by depreciation of 1 per cent., making the talent $\frac{2}{3}$ Babylonian 436,320 grains imperial, and the cubic inch 252·5 grains imperial of rain-water (pp. 81, 90).

## The Mediæval Mark and Pound.

The mark all over Europe was 8 ounces for gold and silver, and the pound was usually 16 ounces for merchandise in Northern Europe, and 12 ounces in parts of Spain and Italy. The pound of 16 ounces was not Avoirdupois weight, *i.e.*, it had a greater number of grains per ounce than 432.

In Kelly's 'Universal Cambist,' 1824, vol. ii., p. 120, will be found tables giving the value of the mark and the 16 ounce pound in the principal towns of Europe and the East.

I tabulate those of Europe, and find that the mark weights fall (with a few abnormal cases) in four groups distinctly accentuated, as follows:

(1.) All France, England, Holland, Western Switzerland, and Northern Italy as far east as Turin.

(2.) All Germany, England, Denmark, East Switzerland, and North-East Italy, including Venice.

(3.) Spain, Portugal, Southern Italy, Rome and Florence.

(4.) Poland, Western Austria and Western Russia.

In group (1) seven cases give an average mark of 3,795 grains imperial, ranging from 3,785 to 3,798 (about ·04 per cent.), giving a 12 ounce pound of 5,692 grains imperial, average and corresponding evidently to the English Troy pound of 5,760 grains imperial (the English weight has not been included in the average).

In group (2) eleven cases give an average mark of 3,645 grains imperial, ranging from 3,608 to 3,643 (about 1 per cent.); this gives a 12 ounce pound of 5,467 grains imperial, corresponding to the English Tower pound of 5,400 grains imperial.

In group (3) seven cases give an average mark of 3,545 grains imperial (ranging from 3,500 to 3,557, or 1·6 per cent.), giving a 12 ounce pound of 5,217, corresponding to the ancient and modern Roman pound of 5,235 grains imperial.

In group (4) several cases give an average mark of 3,195 grains imperial (ranging from 3,023 to 3,113, say 3 per cent.), giving a 12 ounce pound of 4,782 grains imperial, corresponding probably to a weight of 4,860 grains, the Early Roman or Solonian pound.

In Great Britain there are two 12 ounce pounds, viz., the ordinary Troy pound of 5,760 grains imperial, and the Tower pound of 5,400 grains imperial.

Lord Liverpool ('Coins of the Realm,' p. 46) states that up to the reign of King Henry VIII. the Tower pound of 5,400 grains Troy was regarded at the Mint as 5,760 grains: so that

the Anglo-Saxon silver penny of $22\frac{1}{2}$ grains weighed at the Mint 24 grains.

If we take the commercial pound of Europe we find it generally 16 ounces, and to be spread over countries generally in the same manner as the mark, as follows:

| Cases. | | Grains imperial. |
|---|---|---|
| 20 | In the countries under group (1) it represents | 7,680. |
| 30 | „ „ „ (2) „ „ | 7,200. |
| 5 | „ „ „ (3) „ „ | 6,912. |
| 6 | „ „ „ (4) „ „ | 6,480. |

It is thus evident that in the first two cases there were pounds respectively 16 : 15 to each other, the ounces of which would be 480 and 450 grains imperial respectively.

*But* Dr. Clarke ('Connexion of Coins') points out that from the earliest times the pound of Germany brought into England with the Saxons was a 15 *ounce pound*. We thus arrive at the conclusion that the Tower pound (5,400 grains imperial) was originally a pound of 11·25 ounces of 480 grains each. ($11{\cdot}25 \times 480 = 5{,}400$), having the proportion to the Troy pound of 15 : 16.

The Tower pound would also be $\frac{1}{100}$ bushel, if our grain had not depreciated 1 per cent., and if our bushel had not been increased over 1 per cent. in capacity.

If we now examine Table XVI. (p. 128), which I have based on Table XIII. (p. 92), it will be seen that the Troy pound is $\frac{16}{15}$ of the Tower pound.

The interesting question now arises as to when this reversion to the original number 5,760 grains occurred for 12 ounces of 480 grains. So far as our information goes, it is attributed to Charlemagne in the eighth century, but authorities suppose that he may have taken it over from the East.

The Roman pound of 5,235 grains imperial appears to be the only weight in Europe that has absolutely maintained its correctness. It is given in *grains imperial,* which have lost

1 per cent., so that its true weight should be 5,184 grains. (See Table XVI.)

It bears to the Solonian weight the proportion of 16 to 15, and is therefore in the proportion of 9 to 10 to the pound Troy. $\left(5{,}760 \times \frac{9}{10} = 5{,}184.\right)$

It differs about 1 per cent. from 12 ounces Avoirdupois, and does not seem to be connected with the Egyptian system.

Dr. Arbuthnot (1754) tells us that in his day there were 576 grains to the Cairo ounce; the same number as in the Roman, Spanish, and French ounces, but the latter less in weight, and that the number 6,912 grains to the pound is the same in each. In the Roman system we know that as 6,912 grains of wheat = 5,184 grains of barley, the Roman pound = 5,184 grains (corrected Troy). In the Cairo ounce, however, the grain (in 1754) was almost identical with the Troy grain, and this was common to other parts of the East, as, for example at Mocha (Kelly's 'Universal Cambist,' 1824), and in these places also the inch seemed to be identical with the British inch. The Vakia at Mocha is 160 carets (of 3 grains) = 480 grains imperial. It seems probable, then, that there was one value for the grain in weight all over Europe and the East in early historical times, and that we have all derived our standard of length and weight from one and the same source—the $\frac{2}{3}$ (double cubit cubed). Table XVI. gives the changes that have taken place in the weight of the grain. It would appear then that since the earliest times—at least four thousand years ago—the ancient grain of barley has only depreciated in our Troy grain by 1 per cent., the change of $\frac{8}{9}$, or 12·5 per cent., having evidently been carried out by some general decree or agreement. The total depreciation from the ancient grain of barley is 13·5 per cent. for rain-water—a very little less for distilled water. The weights found in the excavations at Jerusalem seem rather to measure Troy grains than ancient grains.

## Table XVI.—Showing Mediæval Weights.

| | | | | | Modern Weights. 252·5 grains to C. I. (rain-water). | | | Mediæval or Earlier. 250 Grains to C. I. | | | Ancient System. $222\frac{2}{9}$ Grains to C.I. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Name of Talent. | Ratio of Contact. | Cubic Foot. C.I. | Cubic Inches. Mina. | Cubic Inches. Pound. | Talent or Cubic Foot. | Mina. | Pound. | Talent or Cubic Foot. | Mina. | Pound. | Talent or Cubic Foot. | Mina. | Pound. |
| Babylonia - | 30 | 2,592·5 | 43·2 | 32·4 | 654,600 | 10,903 | 8181 | 648,000 | 10,800 | 8,100 | 576,000 | 9,600 | 7,200 |
| Euboic - - | 25 | 2,160 | 36 | 27 | 545,400 | 9,090 | 6,817 | 540,000 | 9,000 | 6,750 | 480,000 | 8,000 | 6,000 |
| $\frac{2}{3}$ Babylonian Grecian and British | 20 | 1,728 | 28·8 | 21·6 | 436,320 | 7,272 | 5,454 | 432,000 | 7,200 | 5,400 | 384,000 | 6,400 | 4,800 |
| Mediæval - | $\frac{64}{3}$ | 1,843·2 | 30·7 | 23·0 | 465,408 | 7,756·8 | 5,817·6 | 460,800 | 7,680 | 5,760 | 409,600 | 6,824 | 5,120 |
| Solonian (Early Roman) | 18 | 1,555·2 | 23·9 | 19·4 | 394,760 | 6,546 | 4,908 | 385,000 | 6,480 | 4,860 | 345,600 | 5,760 | 4,320 |
| Late Roman (weights) - | 19.2 | 1,659·18 | 27·6 | 20·7 | 418,544 | 6,981 | 5,235 | 414,400 | 6,912 | 5,184 | 368,640 | 6,144 | 4,608 |

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